FORMULA SHEET

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

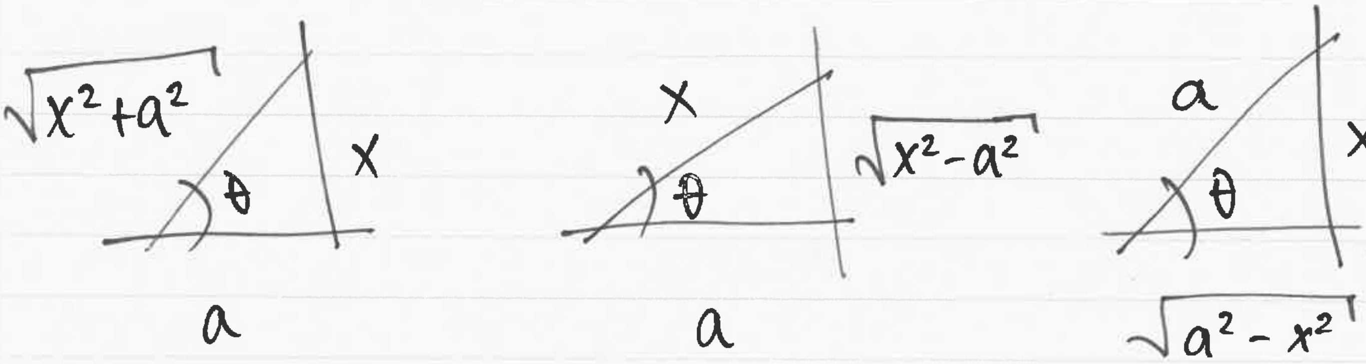
$$\int \sin^2 x \, dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$$\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cancel{\cos 2\theta}$

$\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$ $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

REFERENCE TRIANGLES

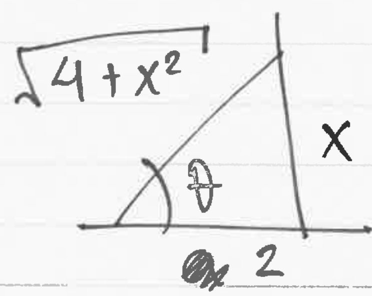


$\int$ TRIG SUBSTITUTION

technique for integrals w/

$$x^2+a^2 \ , \quad x^2-a^2 \ , \quad a^2-x^2.$$

EXAMPLE: $\displaystyle\int \frac{1}{(4+x^2)^{\frac{1}{2}}}\,dx = (*)$



$\frac{x}{2} = \tan\theta \Rightarrow x = 2\tan\theta$

$\Rightarrow dx = 2\sec^2\theta\, d\theta$

$$(*) = \int \frac{2\sec^2\theta}{(4+4\tan^2\theta)^{\frac{1}{2}}}\,d\theta = \frac{2}{2}\int \frac{\sec^2\theta}{(1+\tan^2\theta)^{\frac{1}{2}}}\,d\theta$$

$$= \int \frac{\sec^2\theta}{(\sec^2\theta)^{\frac{1}{2}}}\,d\theta = \int \frac{\sec^2\theta}{|\sec\theta|}\,d\theta = \ln|\sec\theta+\tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

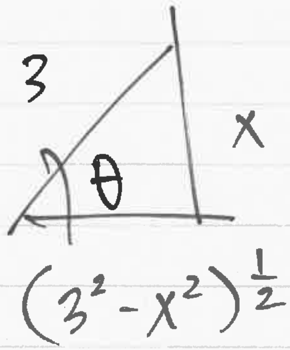
good enough. But "back of textbook" may do

$$= \ln\left|\frac{1}{2}\cdot\left(\sqrt{4+x^2} + x\right)\right| + C$$

$$= \ln\left|\sqrt{4+x^2} + x\right| + \underbrace{\ln\frac{1}{2} + C}_{\text{really just another const.}}$$

$$= \ln\left|\sqrt{4+x^2} + x\right| + D.$$

EXAMPLE: $\displaystyle\int \frac{x^2}{(9-x^2)^{\frac{1}{2}}}\,dx = (*)$



$\frac{x}{3} = \sin\theta \rightarrow x = 3\sin\theta$

$\Rightarrow dx = 3\cos\theta\, d\theta$

$(*) = \displaystyle\int \frac{9\sin^2\theta}{(9-9\sin^2\theta)^{\frac{1}{2}}} \cdot 3\cos\theta\, d\theta$

We need to reduce all the way to $\cos\theta$, $\sin\theta$, $\tan\theta$.

$= \frac{9\cdot 3}{3}\displaystyle\int \frac{\sin^2\theta\cos\theta}{(1-\sin^2\theta)^{\frac{1}{2}}}\, d\theta$

$= 9\displaystyle\int \frac{\sin^2\theta\cos\theta}{(\cos^2\theta)^{\frac{1}{2}}}\, d\theta$

$= 9\displaystyle\int \frac{\sin^2\theta\cos\theta}{|\cos\theta|}\, d\theta$

$= 9\displaystyle\int \sin^2\theta\, d\theta$

$= 9\displaystyle\int \frac{1-\cos 2\theta}{2}\, d\theta$

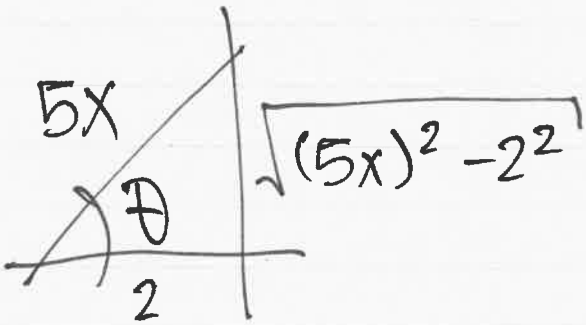
$= \frac{9}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$

$= \frac{9}{2}\left[\theta - \frac{2\sin\theta\cos\theta}{2}\right] + C$

$= \frac{9}{2}\left[\theta - \sin\theta\cos\theta\right] + C$

$= \frac{9}{2}\left[\arcsin\frac{x}{3} - \frac{x}{3}\,\frac{\sqrt{9-x^2}}{3}\right] + C$

EXAMPLE $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}} = \int \frac{1}{\sqrt{(5x)^2-2^2}}\,dx$



$\dfrac{5x}{2} = \sec\theta \Rightarrow 5x = 2\sec\theta$

$\Rightarrow dx = 2/5 \sec\theta \tan\theta \, d\theta$

$$= \int \frac{2/5 \sec\theta \tan\theta}{\left[(2\sec\theta)^2 - 2^2\right]^{\frac{1}{2}}}\,d\theta$$

$$= \frac{2}{2\cdot 5}\int \frac{\sec\theta\tan\theta}{[\sec^2\theta - 1]^{\frac{1}{2}}}\,d\theta$$

$$= \frac{1}{5}\int \frac{\sec\theta\tan\theta}{(\tan^2\theta)^{\frac{1}{2}}}\,d\theta = \frac{1}{5}\int \sec\theta\,d\theta$$

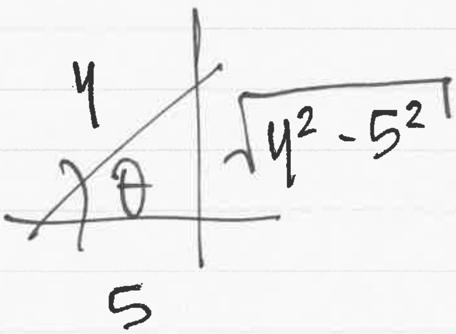
$$= \frac{1}{5}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{5}\ln\left|\frac{5x}{2} + \frac{\sqrt{(5x)^2-2^2}}{2}\right| + C$$

* can you find a mistake? *

I probably made one...

EXAMPLE $\displaystyle\int \frac{(y^2-25)^{\frac{1}{2}}}{y^3}\,dy = (*)$



$$y/5 = \sec\theta \Rightarrow y = 5\sec\theta$$

$$\Rightarrow dy = 5\sec\theta\tan\theta\,d\theta$$

$$(*) = \int \frac{\sqrt{25\sec^2\theta - 25}\cdot 5\sec\theta\tan\theta}{5^3\sec^3\theta}\,d\theta$$

$$= \frac{5.5}{5^3}\int \frac{\sqrt{\sec^2\theta - 1}\cdot\sec\theta\cdot\tan\theta}{\sec^3\theta}\,d\theta$$

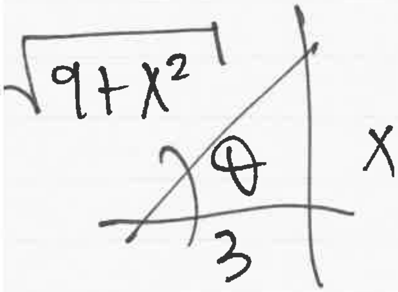
$$= \frac{1}{5}\int \frac{\tan^2\theta\sec\theta}{\sec^3\theta}\,d\theta = \frac{1}{5}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{5}\int \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta\,d\theta = \frac{1}{5}\int \sin^2\theta\,d\theta$$

$$= \frac{1}{5}\cdot\frac{1}{2}\left[\theta - \sin\theta\cos\theta\right]$$

$$= \frac{1}{10}\left[\operatorname{arcsec}\frac{y}{5} - \frac{\sqrt{y^2-25}}{y}\,\frac{5}{y}\right] + C$$

EXAMPLE $\displaystyle\int \frac{1}{\sqrt{9+x^2}}\,dx = (*)$



$\dfrac{x}{3} = \tan\theta \Rightarrow x = 3\tan\theta$

$\Rightarrow dx = 3\sec^2\theta\, d\theta$

$$(*) = \int \frac{3\sec^2\theta}{(9+9\tan^2\theta)^{\frac{1}{2}}}\,d\theta = \int \frac{\sec^2\theta}{(1+\tan^2\theta)^{\frac{1}{2}}}\,d\theta$$

$$= \int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C$$

$$= \ln\left|\sqrt{9+x^2} + x\right| + \ln 3 + C$$