Midterm Review

MATH 134

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Determine whether the statement is true or false. Provide a counter-example if true.

Functions

Question

If f is a function then $f(s+t) = f(s) + f(t)$.

Question

If $f(s) = f(t)$ then $s = t$.

Question

If f is a function, then $f(3x) = 3f(x)$.

Question

If $x_0 < x_1$ and f is decreasing function, then $f(x_0) > f(x_1)$

Question

A vertical line intersects the graph of a function at most once.

Question

If f and g are functions then $f \circ g = g \circ f$.

Question

If f is one-to-one then $f^{-1}(x) = \frac{1}{f(x)}$.

Question

You can always divide by e^x.

Question

If $0 < a < b$ then $\ln a < \ln b$.

Question

If $x > 0$ and $a > 1$ then
$$\frac{\ln x}{\ln a} = \ln \frac{x}{a}.$$

Question

$$\arctan(-1) = \frac{3\pi}{4}$$

Question

$$\arctan x = \frac{\arcsin x}{\arccos x}$$

Limits and Derivatives

Question

If $\lim_{x \to 5} f(x) = 2$ and $\lim_{x \to 5} g(x) = 0$ then $\lim_{x \to 5} \frac{f(x)}{g(x)}$ does not exist.

Question

If $\lim_{x \to 6}[f(x)g(x)]$ exists then $\lim_{x \to 6}[f(6)g(6)]$.

Question

If p is a polynomial then $\lim_{x \to b} p(x) = p(b)$.

Question

If $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$ then $\lim_{x\to 0}[f(x) - g(x)] = 0$.

Question

A function can have two different horizontal asymptotes.

Question

If $\operatorname{dom} f = [0, \infty)$ and has no horizontal asymptotes then $\lim_{x\to\infty} f(x) = \infty$ or $\lim_{x\to\infty} f(x) = -\infty$.

Question

If $x = 1$ is a vertical asymptote of $y = f(x)$ then $1 \notin \operatorname{dom} f$.

Question

If $f(1) > 0$ and $f(3) < 0$ then there exists a number $c \in [1, 3]$ such that $f(c) = 0$.

Question

If f is continuous at 5 and $f(5) = 2$ and $f(4) = 3$ then $\lim_{x \to 2} f(4x^2 - 11) = 2$.

Question

If f is continuous on $[-1, 1$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$.

Question

If $f(x) > 1$ for all x and $\lim_{x \to 0} f(x)$ exists, then $\lim_{x \to 0} f(x) > 1$.

Question

If f is continuous at a then f is differentiable at a.

Question

If $f'(r)$ exists then $\lim_{x \to r} f(x) = f(r)$.

Question

$x^{10} - 10x^2 + 5 = 0$ has a root in $(0, 2)$.

Chapter 3

Question

If f and g are differentiable then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x) + g(x)] = f'(x) + g'(x).$$

Question

If f and g are differentiable then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = f'(x)\,g'(x).$$

Question

If f and g are differentiable then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] = f'(g(x))\,g'(x).$$

Question

If f is differentiable then

$$\frac{\mathrm{d}\sqrt{f(x)}}{\mathrm{d}x} = \frac{f'(x)}{2\sqrt{f(x)}}.$$

Question

If $y = e^2$ then $y' = 2e$.

Question

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln 10) = \frac{1}{10}.$$

Question

If f and g are differentiable then

$$\frac{\mathrm{d}\tan^2 x}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 x).$$

Question

$$\frac{\mathrm{d}}{\mathrm{d}x}|x^2 + x| = |2x + 1|.$$

Question

If $g(x) = x^5$ then $\lim_{x\to 2} \frac{g(x) - g(2)}{x - 2} = 80$.

Question

An equation of the tangent line to the parabola $y = x^2$ at $(-2, 4)$ is $y - 4 = 2x(x + 2)$.

Applications of Differentiation

Question

If $f'(c) = 0$ then f has a local extrema at c.

Question

If f has an absolute minimum at $x = c$ then $f'(c) = 0$.

Question

If f is continuous on (a, b) then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in (a, b).

Question

If f is differentiable and $f(-1) = f(1)$ then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

Question

If $f'(x) < 0$ for $1 < x < 6$ then f is decreasing on $(1, 6)$.

Question

If $f''(2) = 0$ then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

Question

If $f'(x) = g'(x)$ for $x \in (0, 1)$ then $f(x) = g(x)$ for $x \in (0, 1)$.

Question

There exists a function f such that $f(1) = -2$, $f(3) = 0$ and $f'(x) > 1$ for all x.

Question

There exists a function f that $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.

Question

If f and g are increasing on an interval I then $f + g$ is increasing on I.

Question

If f and g are increasing on an interval I then $f \cdot g$ is increasing on I.