§4.8 Antiderivatives

Defn: A function F is the anti-derivative of f when

$$F'(x) = f'(x).$$

We usually denote anti-derivatives by capitals

EXAMPLE

$$f(x) = 2x \Rightarrow F(x) = x^2$$

$$g(x) = \cos x \Rightarrow G(x) = \sin x$$

$$h(x) = \frac{1}{x} + 2e^{2x} \Rightarrow H(x) = \ln x + e^{2x}$$

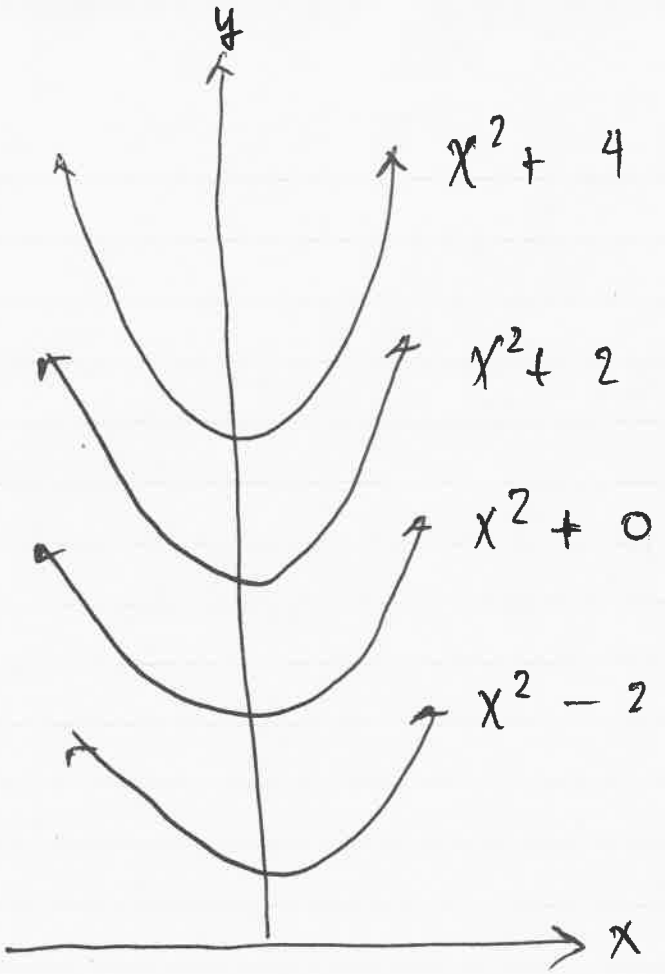
Note these were not the only anti-derivatives.

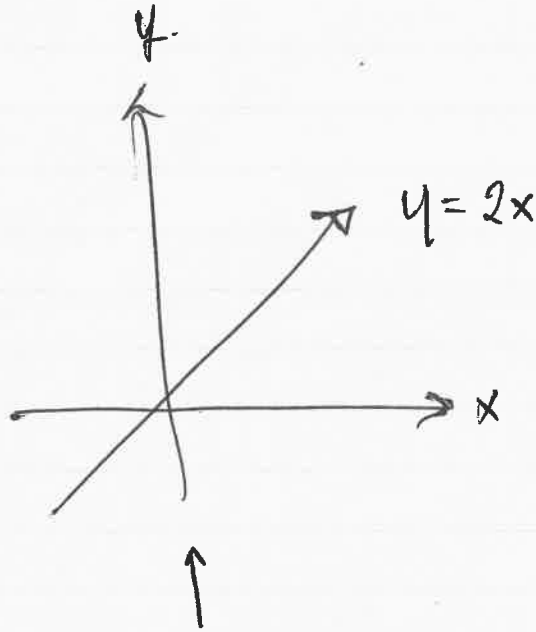
For instance,

$$(x^2 + 7)' = 2x$$

$$(\sin x + 3)' = \cos x$$

So more generally the anti-derivative is a family of functions $F(x) + C : C \in \mathbb{R}$.





all have the same derivative

EXAMPLE: (Initial Value Problem)

Find the anti-derivative F of $f = 3x^2$ such that $F(1) = -1$.

SOLUTION: $F(x) = x^3 + c \quad : \quad c \in \mathbb{R}$

$$\Rightarrow F(1) = -1 = 1^3 + c \Rightarrow c = 0$$

$$F = x^3$$

EXAMPLE Find the "general antiderivative" (hencefourth "antiderivative")

(1.) $f(x) = x^5 \Rightarrow F(x) = \frac{1}{6}x^6 + C : C \in \mathbb{R}$

(2.) $g(x) = \frac{1}{\sqrt{x}} \Rightarrow G(x) = 2\sqrt{x} + C : C \in \mathbb{R}$

(3.) $h(x) = \sin 2x \Rightarrow H(x) = -\frac{1}{2}\cos 2x : c \in \mathbb{R}$

(4.) $i(x) = 2^x \Rightarrow I = 2^x/\ln 2$

Note $\frac{d}{dx} 2^x = 2^x \ln 2$

EXAMPLE $f(x) = 3/\sqrt{x} + \sin 2x$

$\Rightarrow F(x) = 6\sqrt{x} - \frac{1}{2}\cos 2x$

§ Indefinite Integral

Defn Indefinite Integral

the family of anti-derivatives is given by

$$\int f(x)\,dx$$

integral sign — integrand — variable of integration.

EXAMPLE: $\int 2x + 5\,dx = x^2 + 5x + C : C \in \mathbb{R}$.

$$\int \cos x\,dx = \sin x + C : C \in \mathbb{R}$$

EXAMPLE: $\int x^2 - 2x + 5\,dx = x^3/3 - x^2 + 5x + C : C \in \mathbb{R}$

Check your answers by differentiation.

EXERCISE: One of these is harder/impossible atm.

1. $\int x+1\,dx$

2. $\int 3t^2 + \frac{t}{2}\,dx$

3. $\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt$

4. $\int \frac{1}{a} - \frac{b}{x^2}\,dx \quad a,b \in \mathbb{R}.$

5. $\int x\sin x\,dx$

6. $\int x^{\sqrt{2}-1}\,dx$

EXERCISE: One is outright impossible.

1. $\int 3x^{\sqrt{3}}\,dx$

2. $\int 2\cos 2x - 3\sin 3x\,dx$

3. $\int \sin(\cos x)\,dx$

4. $\int \sin 2x - \csc^2 x\,dx$