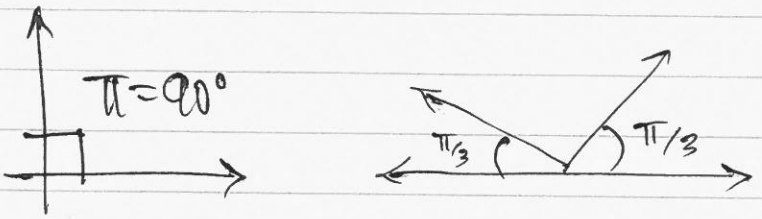
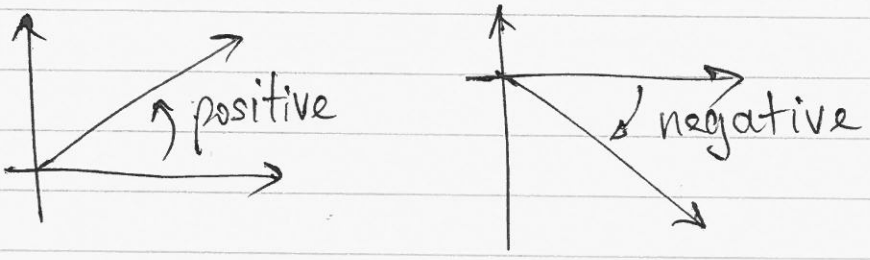
Trigometry

We work in "radians"

$$\pi \text{ rad} = 180^\circ$$

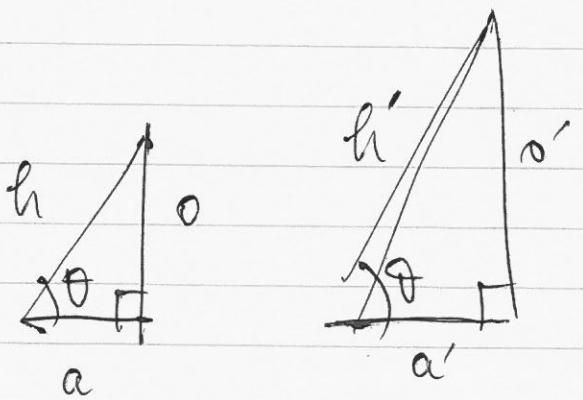


Defn Standard position



Similar triangles share interior angles. The ratio of the sides of similar right-triangles are equal.

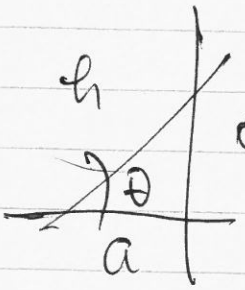
EXAMPLE



$o'/a' = o/a$
$o'/h' = o/h$
$o'/a' = a/a$

If you measure w/ a ruler the triangles above, the ratio of their sides will be equal.

Because similar triangles have this property it is handy to have notation for the various ratios.



$\sin\theta = o/h$ $\quad \csc\theta = h/o$

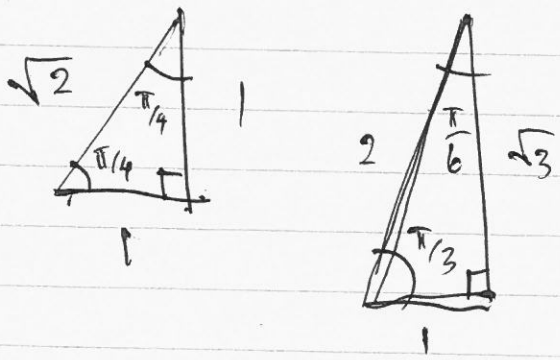
$\cos\theta = a/h$ $\quad \sec\theta = h/a$

$\tan\theta = o/a$ $\quad \cot\theta = a/o$

o = "opposite" a = "adjacent" h = "hypoteneus"

NOTE: $\sin^k\theta$ means $(\sin\theta)^k$.

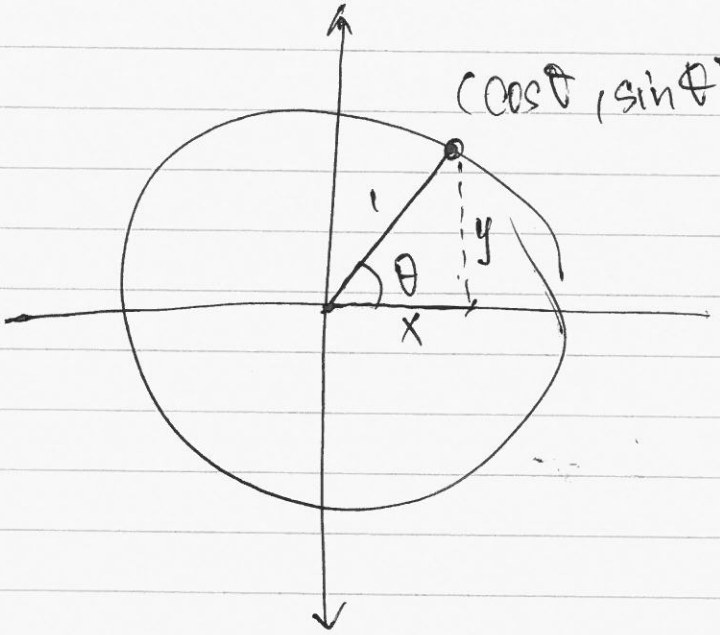
Standard Triangles (Memorize)



Suppose we let $h = 1$ ...

what do the remaining equations express?

All triangles w/ unit hypoteneus lie within the unit circle.



$\cos\theta = x/1 = x$

$\sin\theta = y/1 = y$

$G(\text{unit circle}) = \{(\cos\theta, \sin\theta) : \theta \in [0, 2\pi)\}$

But this implies, via pythagoreus, that

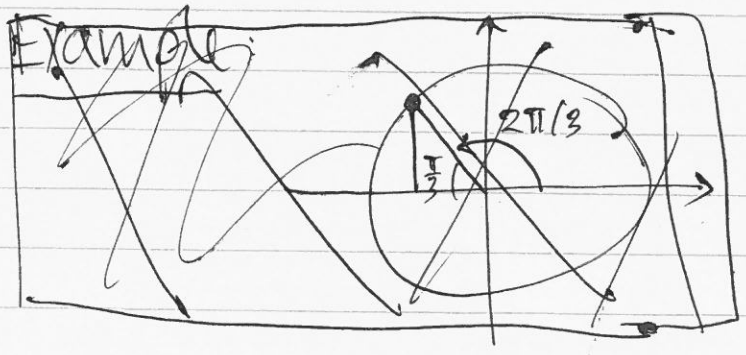
$$\sin^2\theta + \cos^2\theta = 1^2 \quad (*)$$

REFLECT what other rules?

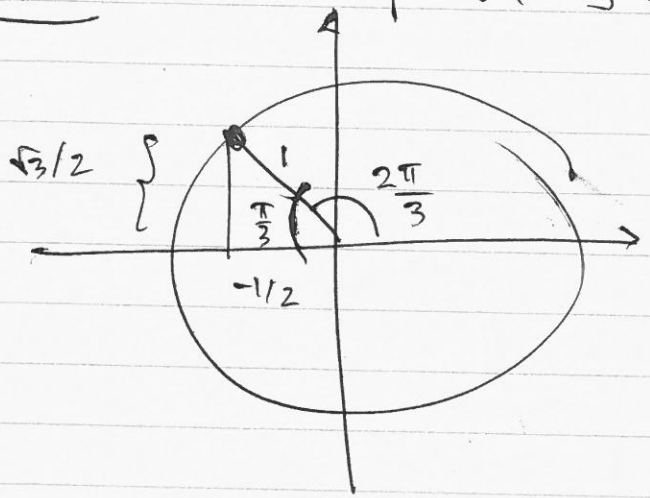
$(*)/\cos^2\theta$: $\tan^2\theta + 1 = \sec^2\theta$

$(*)/\sin^2\theta$: $1 + \cot^2\theta = \csc^2\theta$

Example:



EXAMPLE: Be careful w/ signs.



$\cos\frac{2\pi}{3} = -1/2$ / 1

$\sin\frac{2\pi}{3} = \sqrt{3}/2$ / 1

Notation "$\exists$" there is "$\forall$" for all

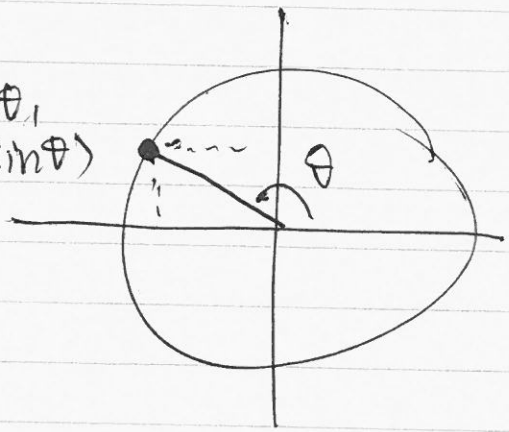
Defn periodic

$f: \mathbb{R} \to \mathbb{R}$ is periodic when

$$\exists p \in \mathbb{R} : f(x+p) = f(x) \ \forall x$$

"there is p in $\mathbb{R}$ such that $f(x+p) = f(x)$ for all x."

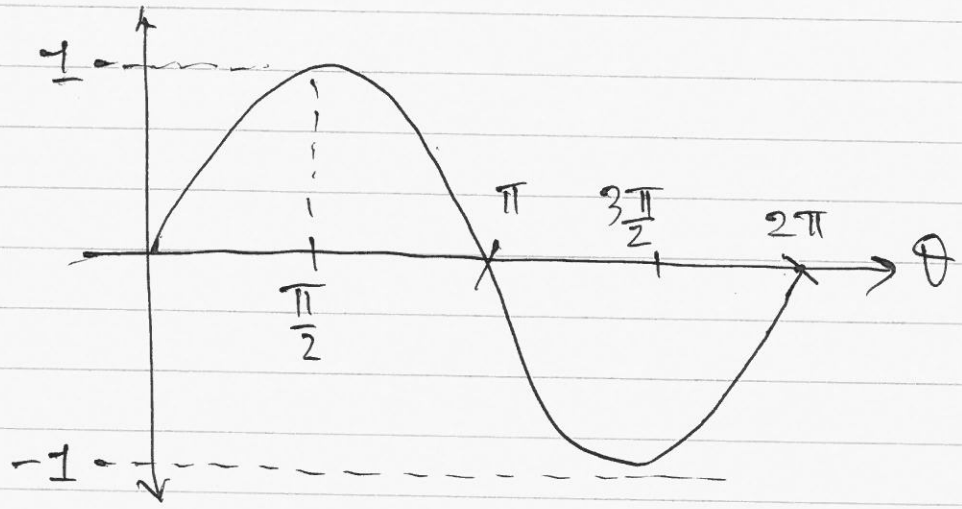
Recall:



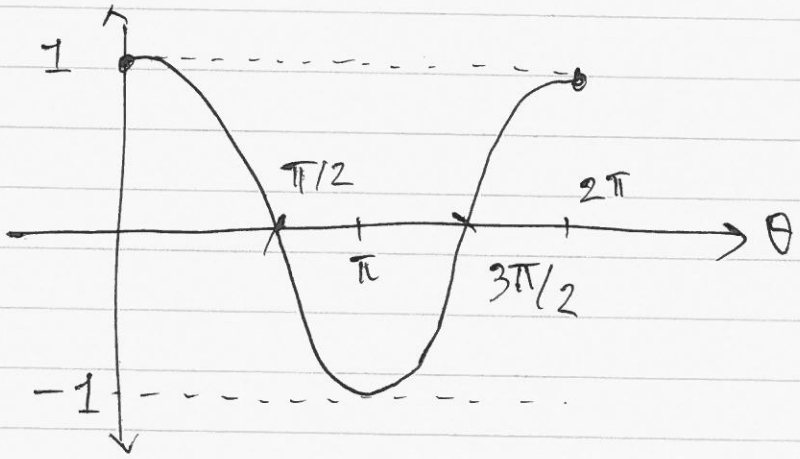
$\cos\theta$ = "how far p is from x-axis"

$\sin\theta$ = "how far p is from y-axis"

$\sin\theta$



$\cos\theta$



Additional Formula

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

let $A = B = \theta \Rightarrow$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$

EXERCISE

$$y = A\sin\left(\frac{2\pi}{B}(x-C)\right)+D$$

Find out what each variable (i.e. Knob) does.

Initially let $A=1$, $B=2\pi$, $C=0$, $D=0$.

QUESTION: Find a function that crosses the ~~x-values~~ x-axis at integer values only.

~~ANSWER: $\sin(x \cdot 2\pi) = f(x)$~~

ANSWER: $\sin(\pi x) = f(x)$

EXERCISE: Solve $|x-2| = 10$ "algebraically"

Then geometrically using desmos.

§2.3 EXPONENTIAL FUNCTIONS

Important for exponential growth.

Defn Exponential Function

$f: \mathbb{R} \to \mathbb{R}$ and

$f(x) = Ka^x$ for $K \in \mathbb{R}$.

Defn "the base of the natural log"

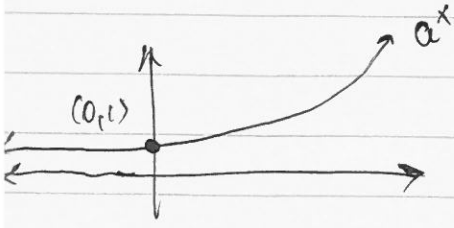
$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots \approx 2.72$$

Question Find $y = a^x$ that passes through $(3, 8)$
G2-3.1

Question G.2-3-3: Find the y-int of $f(x) = 2\left(\frac{1}{2}\right)^x$

Question G.2-3-4: Let $y = 1/e + e^{1/x}$. Find dom y.

Example: $a > 1$



$a < 1$