Algorithmic Complexity

Introduction to Computer Programming

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Definition (Complexity)

We measure the complexity of an algorithm by counting the number of operations performed as a function of the length of the input (usually denoted n).

Linear

```
def linear(L: List[int]):
    n = len(L)                  # 1
    index = 0                   # 1
    while index < len(L):       # n
        index += 1              # n
                                # 2n + 1 = O(n)
```

Definition (Big-Oh Notation)

We say that "$f(n)$ is big-oh of $g(n)$" and write $f(n) = \mathcal{O}(g(n))$ when

$$\exists\, c,\, k \in \mathbb{R}^{>0} : \forall n \geq k;\, 0 \leq f(n) \leq cg(n).$$

Example

1. $2x^2 + x + 1 = \mathcal{O}(n^2)$ because for $x > 2$
$$2x^2 + x + 1 < 3x^2.$$

2. $x^3 - \sin(x) = \mathcal{O}(n^3)$ because for $x > 1$
$$x^3 - \sin(x) \leq x^3 + 1 < 2x^3.$$

Question

What is the complexity of `foo`?

```
def foo(L: List[int]):
    n = len(L)
    index = 0
    while index < len(L):
        index2 = 0
        while index2 < len(L):
            index2 += 1
        index += 1
```

Quadratic

```
def quadratic(L: List[int]):
    n = len(L)
    index = 0                          # 1
    while index < len(L):              # n
        index2 = 0                     # n
        while index2 < len(L):         # n * n
            index2 += 1                # n * n
        index += 1                     # n
                                       # 2n**2 + 3n + 1 = O(n**2)
```

Question

What is the complexity of `foo`?

```
def foo(L: List[int]):
    n = len(L)
    index = 0
    while i < 10**10:
        index += 1
```

Constant

```
def foo(L: List[int]):
    n = len(L)              # 1
    index = 0               # 1
    while i < 10**10:       # 10**10
        index += 1          # 10**10
                            # 2*10**10 + 2 = O(1)
```

Question

What is the complexity of `foo`?

```
def foo(L: List[int]):
    n = len(L)
    index = 0
    while 2 ** index < len(L):
        index += 1
```

Logarithmic

```
def log(L: List[int]):
    n = len(L)                          # 1
    index = 0                           # 1
    while 2 ** index < len(L):          # log[2](n)
        index += 1                      # log[2](n)
                                        # 2log[2](n) + 2 = O(ln(n))
```

Question

What is the algorithmic of the following algorithms as measured by the length of the input list?

1. Selection,
2. Insertion, and
3. Bubble.

Answer

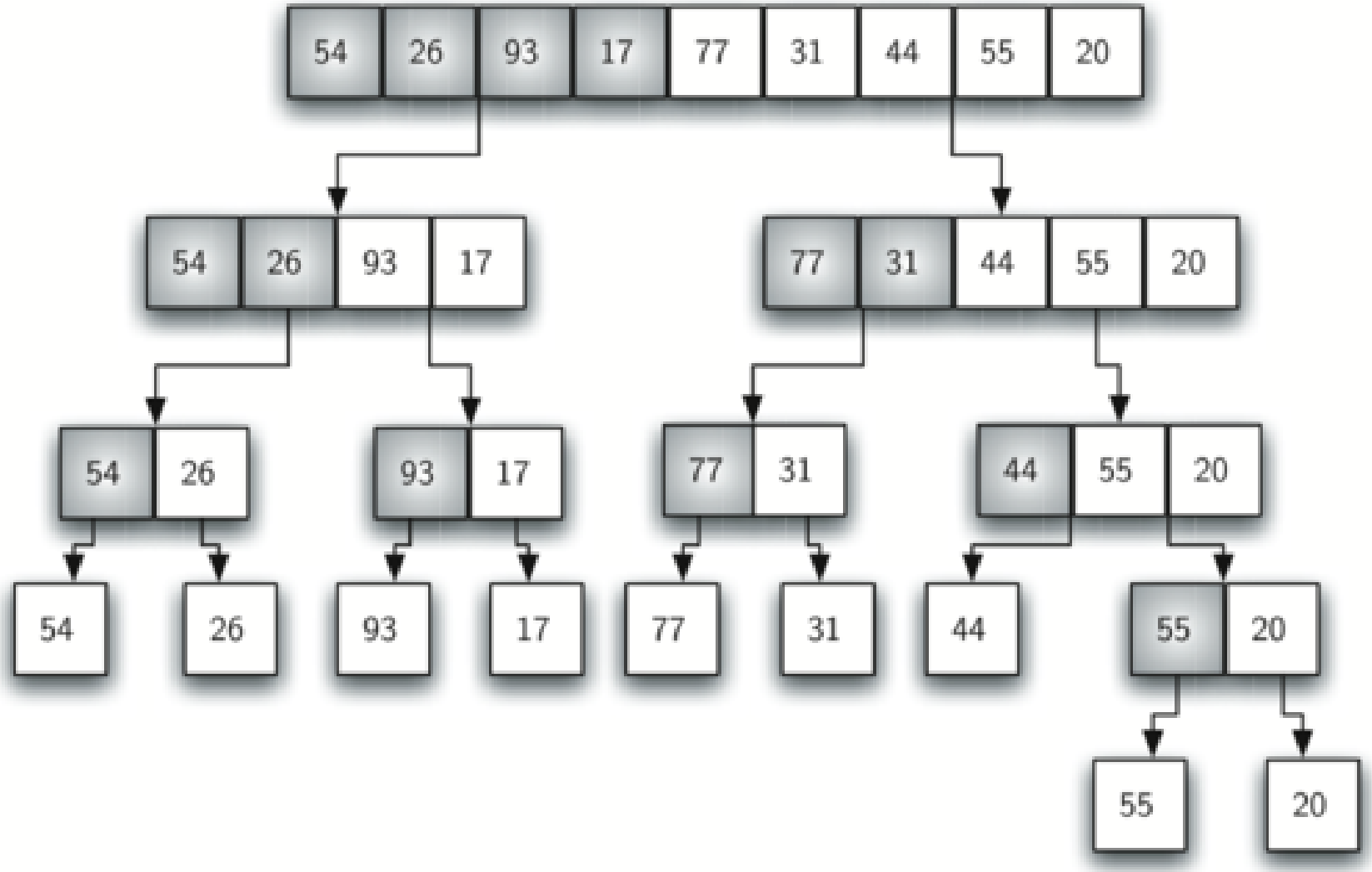
$\mathcal{O}(n^2)$.

Question

Suppose the only operation we could perform on lists was merging two sorted lists into one sorted list. Could use this to sort a list?

Merge Sort

Break the list in half and sort the halves (using merge sort) then combine these pieces into a single sorted list being the answer.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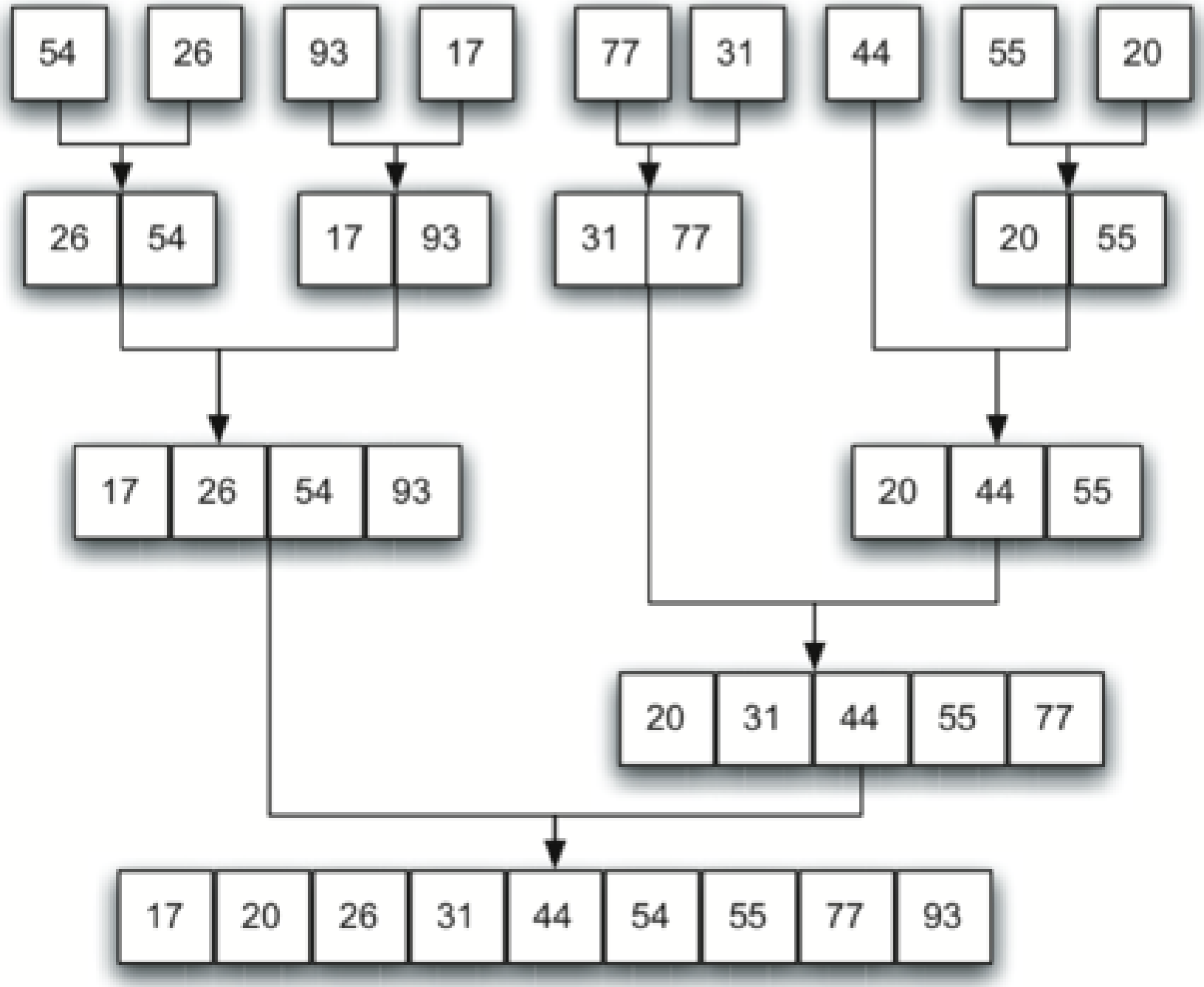
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