Assignment 3

October 20, 2009

1. Give the steps of the extended Euclidean algorithm with inputs $A_0 = 3 + x - x^2 + x^3$ and $A_1 = 1 - x + x^2$.

2. The *golden ratio* is $\varphi = (1 + \sqrt{5})/2$.

   - Find a polynomial P of degree 2 such that $P(\varphi) = 2$
   - Use Euclidean division and XGCD computation to express $(\varphi - 1)/(\varphi + 2)$ in the form
   $$a_0 + a_1\varphi,$$
   with a_0, a_1 rational numbers.
   - Could you do the same for more general fractions in φ? How?

3. Consider the sequence $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots)$. Using the extended Euclidean algorithm, find a linear recurrence of order 2, with constant coefficients, that is satisfied by this sequence.

4. Given points $a_1, \dots, a_n$ and values $v_1, \dots, v_n$, with $a_i \neq a_j$ for $i \neq j$, we want to find a rational function $P = N/D$, with $\deg(N) < n/2$ and $\deg(D) = n/2$ such that $P(a_i) = v_i$ for all i.

   Suppose that we know a polynomial $Q(x)$ of degree less than n such that $Q(a_i) = v_i$ for all i. Let also $M = (x - a_1) \cdots (x - a_n)$. Using the same idea as for the rational function reconstruction seen in class, explain how you could solve your problem by applying the XGCD algorithm to Q and M.

   I'm not asking for a detailed proof, but for the idea of the algorithm, and an informal justification of why it will work.

5. How much time did you spend on the assignment?