CS 3331a - Assignment 4 - Solutions

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Question 1 - 20 - marks

Give a regular grammar for each of the following languages:

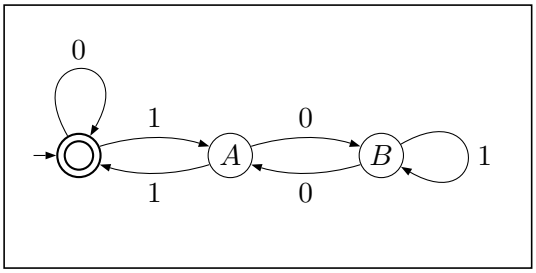
(1) $L_1 = \{0^i 1^j | i, j \geq 0\}$.

$$S \to \varepsilon \mid A \mid 0S$$
$$A \to \varepsilon \mid 1A$$

(2) L_2 is the set of all words over $\Sigma = \{a, b\}$ of even length.

$$S \to \varepsilon \mid aA \mid bA$$
$$A \to aS \mid bS$$

(3) L_3 is the set of all binary numbers divisible by 3. (*Note: L_3 is generated by following DFA.*)



$$S \to \varepsilon \mid 0S \mid 1A$$
$$A \to 0B \mid 1S$$
$$B \to 1B \mid 0A$$

Question 2 - 20 marks

Change the CFG into an equivalent CFG in Chomsky normal form:

1. Reduction

Nonterminals = $\{C\}$

Unreachable = $\{E, F\}$

$S \to aAB \mid \cancel{bBBBC}$

$A \to aB \mid D \mid \varepsilon$

$D \to AB$

$B \to bA \mid \varepsilon$

$\cancel{C} \to aC$

$\cancel{E} \to cB$

$\cancel{F} \to bAA \mid \varepsilon$

2i. Remove ε-productions

ε-nonterminals = $\{A \to \varepsilon, B \to \varepsilon\}$

Remove $A \to \varepsilon$

$S \to aAB \mid aB$

$A \to aB \mid D$

$D \to AB \mid B$

$B \to bA \mid b \mid \varepsilon$

2ii. Remove $B \to \varepsilon$

$S \to aAB \mid aA \mid aB \mid a$

$A \to a \mid aB \mid D$

$D \to AB \mid A \mid B$

$B \to bA \mid b$

3. Remove Unit $A \to D$ and Reduce

$S \to aAB \mid aA \mid aB \mid a$

$A \to a \mid aB \mid AB \mid aB \mid bA \mid b$

$B \to bA \mid b$

4. Remove Long production $S \to aAB$

$S \to aT \mid aA \mid aB \mid a$

$T \to AB$

$A \to a \mid aB \mid AB \mid aB \mid bA \mid b$

$B \to bA \mid b$

5. Convert to Chomsky normal form

$S \to \bar{a}T \mid \bar{a}A \mid \bar{a}B \mid \bar{a}$

$T \to AB$

$A \to \bar{a} \mid \bar{a}B \mid AB \mid \bar{a}B \mid \bar{b}A \mid \bar{b}$

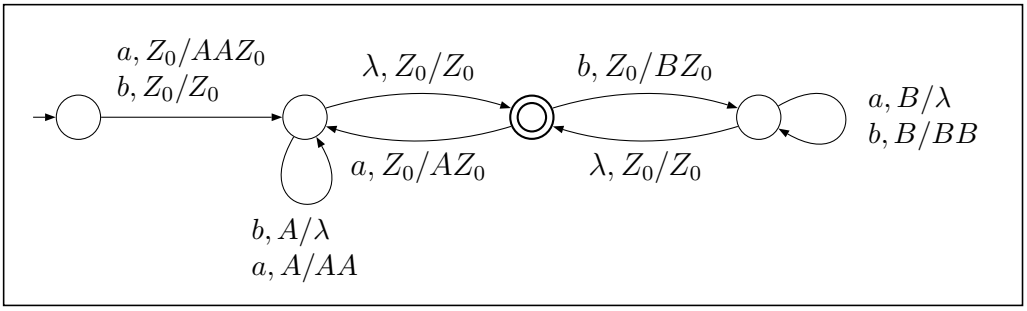
$B \to \bar{b}A \mid \bar{b}$

$\bar{a} \to a$

$\bar{b} \to b$

Question 3 - 20 marks

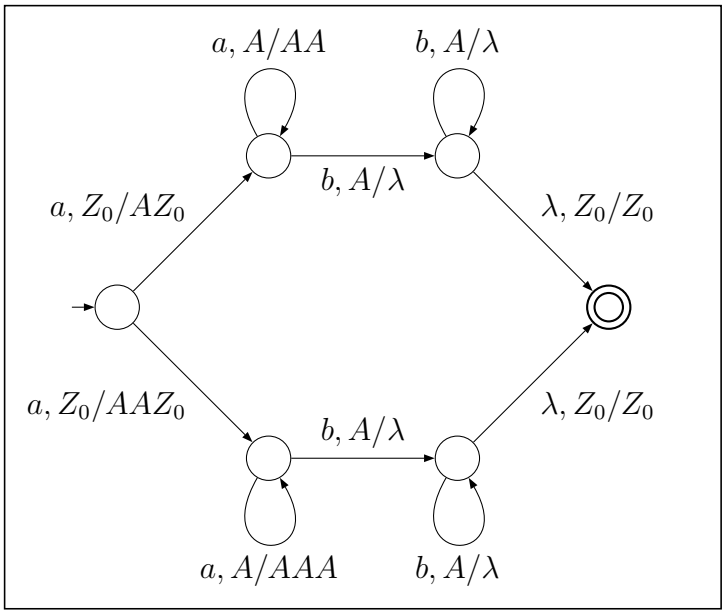
Construct a *deterministic* pushdown automaton that accepts all words in $\{a, b\}^*$ such that the number of a's is one less than the number of b's. (Both numbers are greater than or equal to zero.)



Question 4 - 20 marks

Construct a pushdown automaton to accept the language $\{a^i b^i | i > 0\} \cup \{a^i b^{2i} | i > 0\}$.

The machine below accepts the required language by "final state".



Question 5 - 20 marks

Use the pumping lemma for context-free languages to show that $L = \{ww | w \in \{0, 1\}^*\}$ is *not* a context-free language.

This question is precisely Example 7.21 of the textbook (p. 285 or p. 279). As a full solution is provided there I will omit it here.

Question 6 (Bonus) - 10 marks

Give a context-free grammar that generates the set of all words, over the alphabet $\Sigma = \{a, b\}$, that are not of the form ww for any word $w \in \Sigma^*$.

$$
\begin{aligned}
S &\to A \mid B \mid AB \mid BA \\
A &\to CAC \mid a \\
B &\to CBC \mid b \\
C &\to a \mid b
\end{aligned}
$$

Good luck on your exam everyone! Have a great holiday!

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