CS 3331a - Assignment 4 - Solutions

Paul Vrbik

November 30, 2011

Question 1

Give a CFG for each of the following languages:

(1) $L_1 = \{a^i b^j \mid i \neq j, \ i, j \geq 0\}$

$$N = \{S, A, B\}, \ \Sigma = \{a, b\}$$
$$P : S \to aSb \mid A \mid B$$
$$A \to a \mid aA$$
$$B \to b \mid Bb$$

(2) $L_2 = \{a^i b^j \mid i \leq j \leq 2i\}$

$$N = \{S\}, \ \Sigma = \{a, b\}$$
$$P : S \to \varepsilon \mid aSb \mid aSbb$$

(3) The set of all strings over alphabet $\{a, b, \cdot, +, *, (,), \varepsilon, \varnothing\}$ that are well-formed regular expressions over the alphabet $\{a, b\}$.

$$N = \{S\}, \ \Sigma = \{a, b, \cdot, +, *, (,), \varepsilon, \varnothing\}$$
$$P : S \to \varnothing \mid \varepsilon \mid a \mid b \mid S + S \mid S \cdot S \mid S^* \mid (S)$$

(4) $L = \{a^{i+3} b^{2i+1} \mid i \geq 0\}$

$$N = \{S\}, \ \Sigma = \{a, b\}$$
$$P : S \to aaab \mid aSbb$$

Question 2

Change the CFG into an equivalent CFG in Chomsky normal form:

1. Reduction

1i. Reduction : remove Nonterminating $= \{E\}$

$$S \to b\!\!\!/\!E \mid aAC$$
$$A \to aB \mid D \mid \varepsilon$$
$$D \to bAB$$
$$B \to b \mid \varepsilon$$
$$C \to c$$
$$\not{E} \to cE$$
$$F \to ABC \mid \varepsilon$$

1ii. Reduction : remove Nonreachables $= \{F\}$

$$S \to aAC$$
$$A \to aB \mid D \mid \varepsilon$$
$$D \to bAB$$
$$B \to b \mid \varepsilon$$
$$C \to c$$
$$\not{F} \to ABC \mid \varepsilon$$

2. Remove ε-productions

2i. Remove $A \to \varepsilon$

$$S \to aAC \mid \boldsymbol{aC}$$
$$A \to aB \mid D \mid \not{\varepsilon}$$
$$D \to bAB \mid \boldsymbol{bB}$$
$$B \to b \mid \varepsilon$$
$$C \to c$$

2ii. Remove $B \to \varepsilon$

$$S \to aAc \mid aC$$
$$A \to aB \mid \boldsymbol{a} \mid D$$
$$D \to bAB \mid \boldsymbol{bA} \mid bB \mid \boldsymbol{b}$$
$$B \to b \mid \not{\varepsilon}$$
$$C \to c$$

3. Remove Units

Remove $A \to D$

$$
\begin{aligned}
S &\to aAc \mid aC \\
A &\to aB \mid a \mid \boldsymbol{bAB} \mid \boldsymbol{bA} \mid \boldsymbol{bB} \mid \boldsymbol{b} \\
D &\to bAB \mid bA \mid bB \mid b \\
B &\to b \\
C &\to c
\end{aligned}
$$

Reduce (D now unreachable).

$$
\begin{aligned}
S &\to aAc \mid aC \\
A &\to aB \mid a \mid bAB \mid bA \mid bB \mid b \\
B &\to b \\
\cancel{D} &\to bAB \mid bA \mid bB \mid b \\
C &\to c
\end{aligned}
$$

3. Remove Long Productions

$$
\begin{aligned}
S &\to a\boldsymbol{T} \mid aC \\
\boldsymbol{T} &\to Ac \\
A &\to aB \mid a \mid b\boldsymbol{U} \mid bA \mid bB \mid b \\
\boldsymbol{U} &\to AB \\
B &\to b \\
C &\to c
\end{aligned}
$$

4. Convert to Chomsky Normal Form

$$
\begin{aligned}
S &\to \bar{a}T \mid \bar{a}C \\
T &\to AC \\
A &\to \bar{a}B \mid a \mid BU \mid BA \mid BB \mid b \\
U &\to AB \\
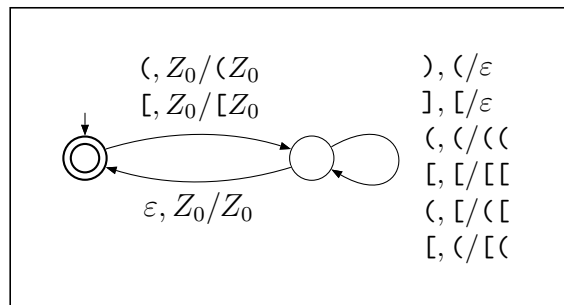B &\to b \\
C &\to c \\
\bar{a} &\to a
\end{aligned}
$$

The following is also OK.

$$S \to \bar{a}T \mid \bar{a}C$$
$$T \to A\bar{c}$$
$$A \to \bar{a}B \mid a \mid \bar{b}U \mid \bar{b}A \mid \bar{b}B \mid b$$
$$U \to AB$$
$$B \to b$$
$$C \to c$$
$$\bar{a} \to a$$
$$\bar{b} \to b$$
$$\bar{c} \to c$$

Question 3

Construct a deterministic pushdown automaton that accepts the set of all words of balanced parentheses and square brackets.
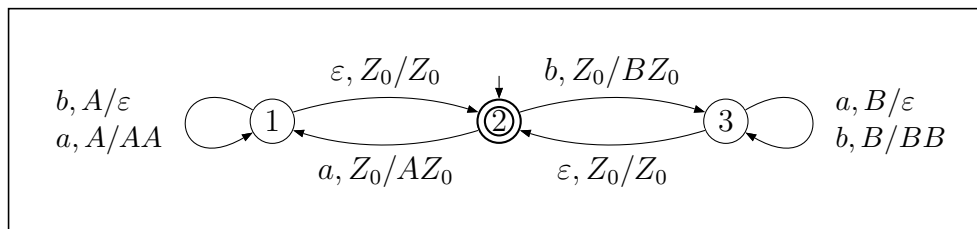
The machine below accepts the required language by "final state".

Question 4

Construct a pushdown automaton that accepts all words in $\{a, b\}^*$ that contain an equal number of a's and b's.

The machine below accepts the required language by "final state".

Note, in the above PDA: State 1 represents having read more a's than b's, State 2 an equal amount of a's and b's, and State 3 more b's than a's.

Good luck on your exam everyone! Have a great holiday!