CS 3331a - Assignment 3 - Solutions

Paul Vrbik

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Question 1 - marks

Write a regular expression for the following languages over $\{0, 1\}$:

(1) the set of words that start with 1, end with 11, and have 010 as a subword.

$$(1)(0+1)^*(010)(0+1)^*(11)$$

(2) the set of all words not containing consecutive 1's.

$$(10+0)^*(1+\varepsilon) \text{ or } (1+\varepsilon)(01+0)^*$$

Question 2 - marks

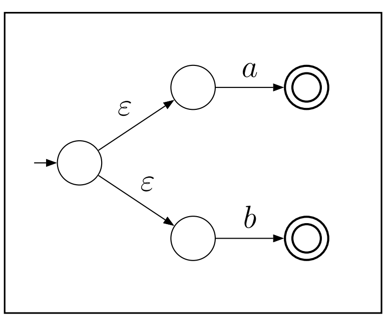
Given the following regular expression E,

$$(a+b)^*ab^*a(a+b)^*$$

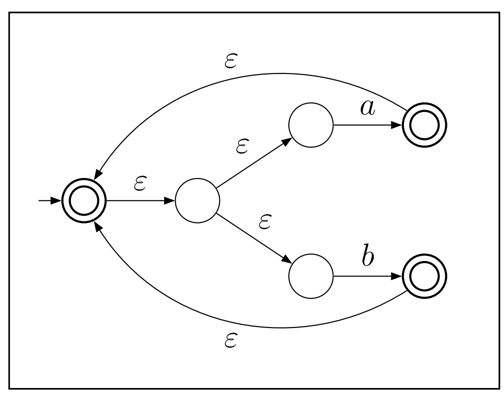
construct an ε-NFA A such that $L(A) = L(E)$.

I follow the procedure of the notes and not that of the textbook.

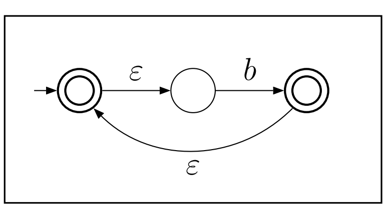
$(a+b)$:



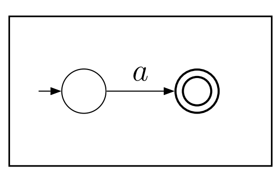
$(a+b)^*$:



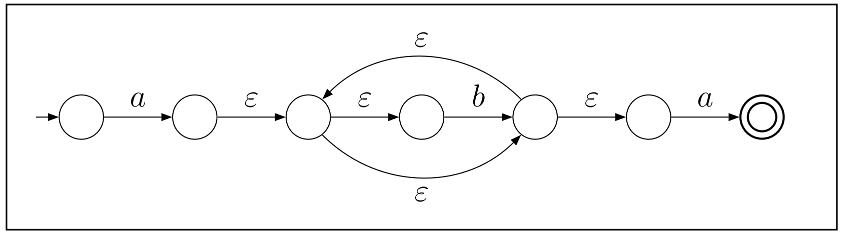
b^* :



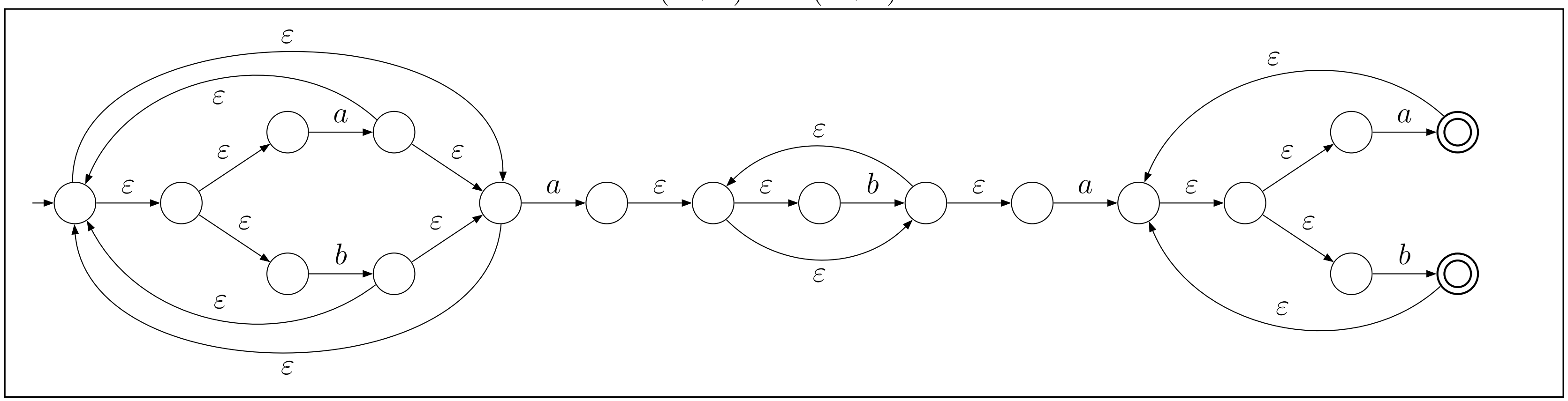
a :



ab^*a :



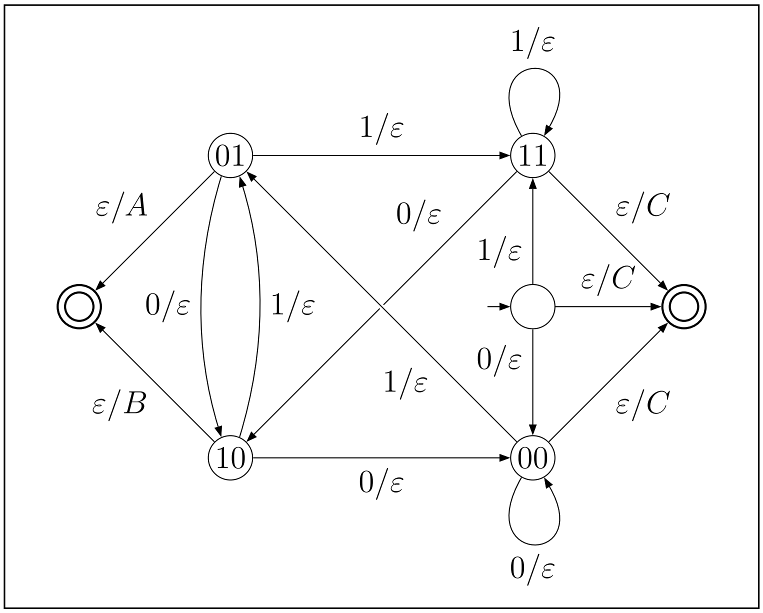
$(a+b)^*ab^*a(a+b)^*$



Question 3 - marks

Construct a finite transducer for the following process:

For input from $\{0,1\}^*$, if the input ends in 01, output A; if the input ends in 10, output B; otherwise, output C.



Question 4 (Bonus) - marks

Let L_1 and L_2 be two languages. Define

$$L_2 \setminus L_1 = \{y \mid xy \in L_1 \text{ and } x \in L_2\}.$$

Prove that if L_1 is a DFA language, then $L_2 \setminus L_1$ is a DFA language.

Proof. Let $M = \{Q, \Sigma, \delta, s, F\}$ be a DFA such that $L(M) = L_1$ and L_2 any language.

WLOG assume $s_0 \notin Q$. Consider the machine $N = \{Q \cup \{s_0\}, \Sigma, \delta', s_0, F\}$ with

1. $\delta'(r,a) = \delta(r,a)$ for any $(x,y) \in Q \times \Sigma$, and
2. $\delta'(s_0,\varepsilon) = q \in Q$ only when there is $x \in L_2 \cap L(M_q)$ for $M_q = \{Q, \Sigma, \delta, s, \{q\}\}$.

That is, q is made an initial state of N (more accurately: an ε-transition is made from some common initial state to q) if when q is made a final state of M, some word of L_2 is accepted by M.

Now consider any word $xy \in L_1$ such that $x \in L_2$ with accepting configuration

$$sxy \vdash \cdots \vdash ty \vdash \cdots \vdash u$$

in M with $u \in F$. By our construction t is an "initial" state of N and so it follows $ty \vdash \cdots \vdash u$ is an accepting configuration in N. Thus $y \in L(N)$ and it is proved that $L_2 \setminus L_1 \subseteq L(N)$.

Suppose $y \in L(N)$ has accepting configuration

$$s_0 \varepsilon y \vdash s_i y \vdash \cdots \vdash u$$

$s_i \in Q$ and $u \in F$. Since $s_i \neq s_0$ there must exist some nonempty word x for which $sx \vdash \cdots \vdash s_i$ in M. By our construction s_i is "initial" in N only when $x \in L_2$; moreover

$$sxy \vdash \cdots \vdash s_i y \vdash \cdots \vdash u$$

in M, putting $xy \in L_1$. Thus $L(N) \subseteq L_2 \setminus L_1$.

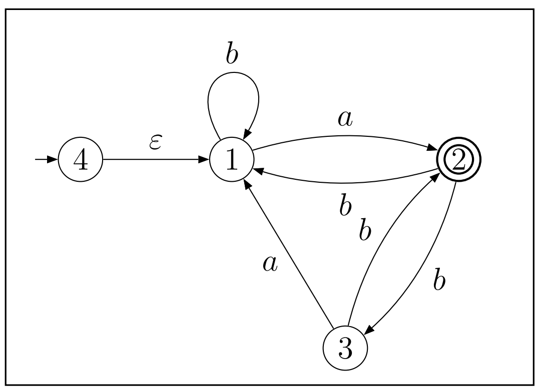
As the ε-NFA N can be converted to a DFA, N', it follows that $L_2 \setminus L_1$ is a DFA language.

□

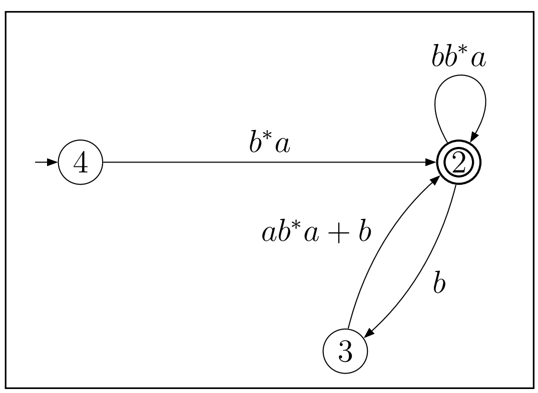
Question 5 - marks

Given the following DFA, construct an equivalent regular expression.

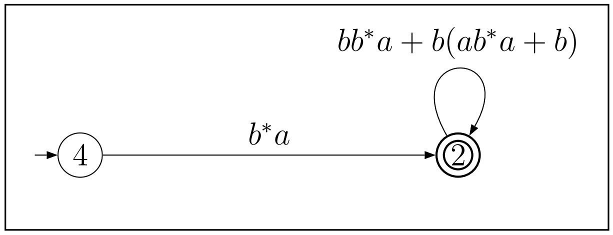
- Add a different initial state.



- Eliminate State 1



- Eliminate State 3



- Combine remaining state transition rules