CS 3331a - Assignment 2 - Solutions

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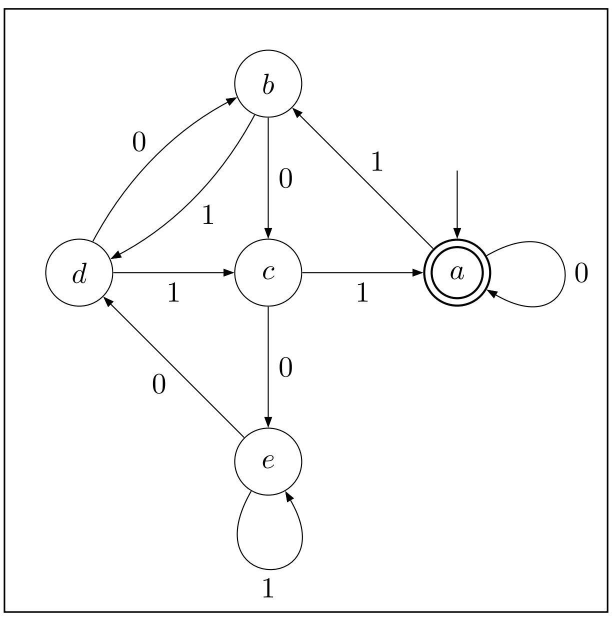
Question 1 - marks

Give a full definition of a deterministic finite automaton (DFA) that accepts the set of all binary numbers (over the alphabet $\{0, 1\}$ each of which has a value divisible by 5. (Note that the set includes $\varepsilon, 101, 1010, 1111, \ldots$.)

Solution.

Note : in the diagram below states a, b, c, d and e denote 0 mod 5, 1 mod 5, 2 mod 5, 3 mod 5, and 4 mod 5 (respectively).



Let us call our DFA A. Using the "five-tuple" notation we have:

$$A = (\{a, b, c, d, e\}, \{0, 1\}, \delta, a, \{a\})$$

where $\delta : Q \times \Sigma \to Q$ is given by

$$\begin{array}{lllll} \delta(a,0) = a & \delta(b,0) = c & \delta(c,0) = e & \delta(d,0) = b & \delta(e,0) = d \\ \delta(a,1) = b & \delta(b,1) = d & \delta(c,1) = a & \delta(d,1) = c & \delta(e,1) = e\,. \end{array}$$

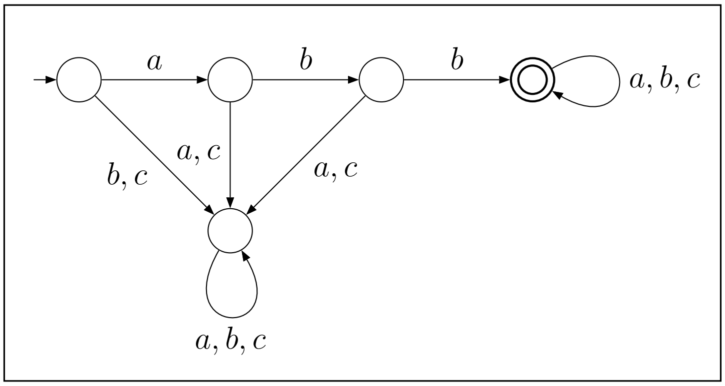
□

Question 2 - marks

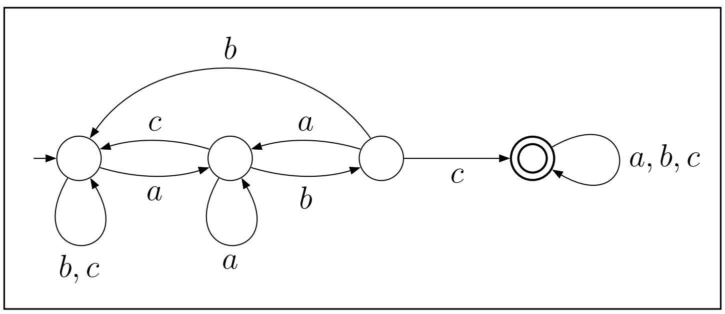
Give a deterministic finite automata (DFAs) accepting the following languages over the alphabet $\{a, b, c\}$. Note that all the DFAs are required to be complete DFAs. (Transition diagrams only).

Solution.

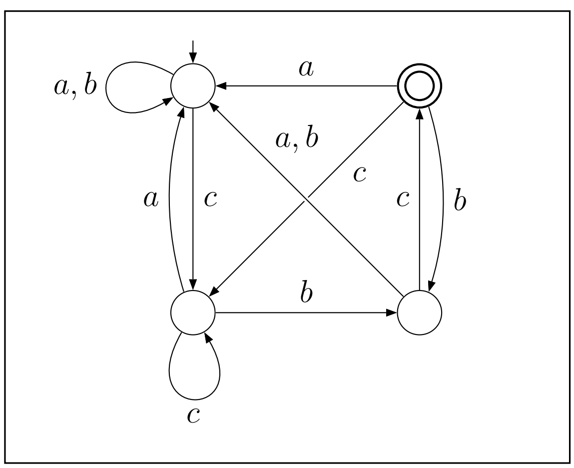
(1) The set of all words that have *abb* as a prefix.



(2) The set of all words that have *abc* as a subword.

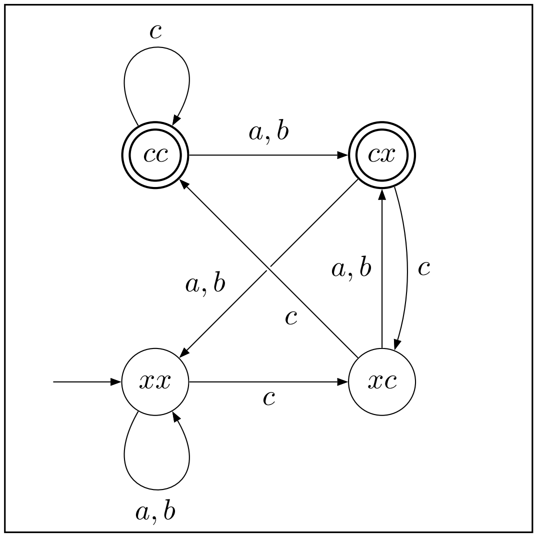


(3) The set of all words ending in *cbc*.



(4) The set of all words such that the second symbol from the right-end is c.

There is a strategy which makes this question easier: create states for all possible length two substrings (there are nine of them — but since we don't care to distinguish 'a' and 'b' we can represent both with 'x' and reduce to four substrings) and draw rules for transitioning between them. The accepting states are those substrings that begin with 'c'. (I've labelled the states below to reflect this.) You may interpret the start state as being 'bb' (you could also pick 'aa') because at least two moves to get to an accepting state from there.



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Question 3 - marks

Prove that the language $L = \left\{ a^i b^j \,\middle|\, 0 \le j < i \right\}$ is not accepted by *any* DFA.

Proof. Towards a contradiction assume there is a DFA

$$M = (Q, \{a, b\}, \delta, s, F)$$

such that $L = L(M)$. Let $n = \#Q$ (the number of states). Consider the accepting configuration for $a^n b^{n-1}$,

$$\underbrace{s_0 a^n b^{n-1} \vdash s_1 a^{n-1} b^{n-1} \vdash \cdots \vdash s_n b^{n-1}}_{n+1 \text{ states required}} \vdash \cdots \vdash s_{2n-1}$$

where $s_0 = s$ and $s_n \in F$. Note that $n+1$ states are required to read a^n (as indicated). However, by our assumption there is only n states; therefore by Pigeonhole principle two of these states must be the same. That is, there is $s_i = s_j$ such that $0 \le j < i \le n$.

Now consider the accepting configuration,

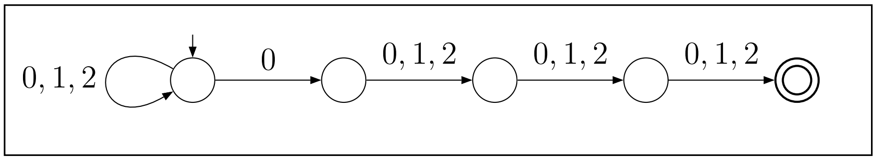
$$\begin{aligned} s_0 a^{n+(j-i)} b^{n+1} \vdash \cdots \vdash s_j a^{n-i} b^{n+1} & \\ \vdash s_i a^{n-i} b^{n+1} \vdash \cdots \vdash s_{2n+1} & \end{aligned}$$

which shows that $a^{n+(j-i)} b^{n-1} \in L$ as well. Since $j - i < 0$ this is a contraction. ∎

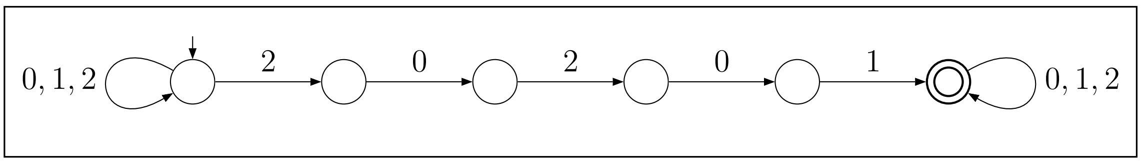
Question 4 - marks

Design nondeterministic finite automata (NFA) for the following languages over the alphabet $\{0, 1, 2\}$. (Transition diagrams only).

(1) The set of all words such that the fourth symbol from the right-end is 0.



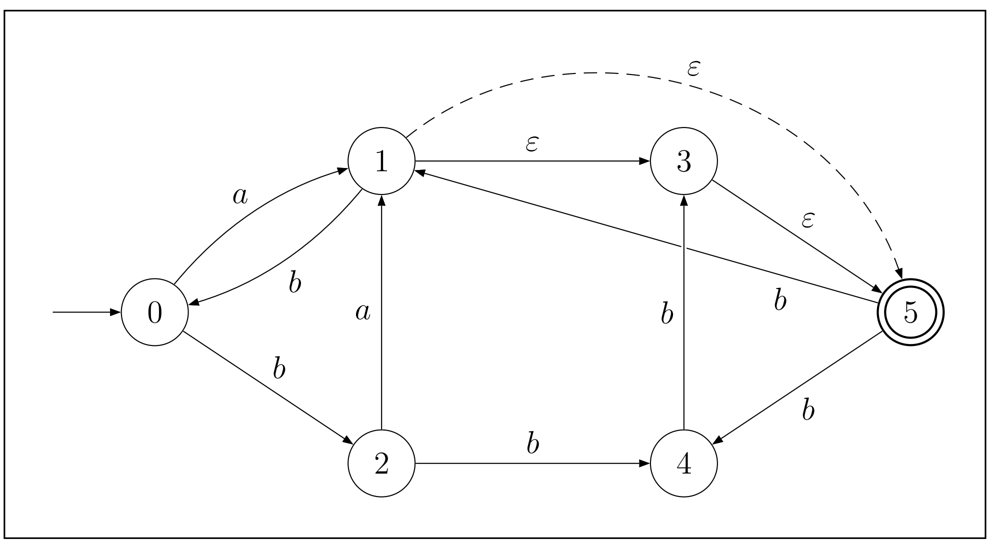
(2) The set of all words that have a subword 20201.



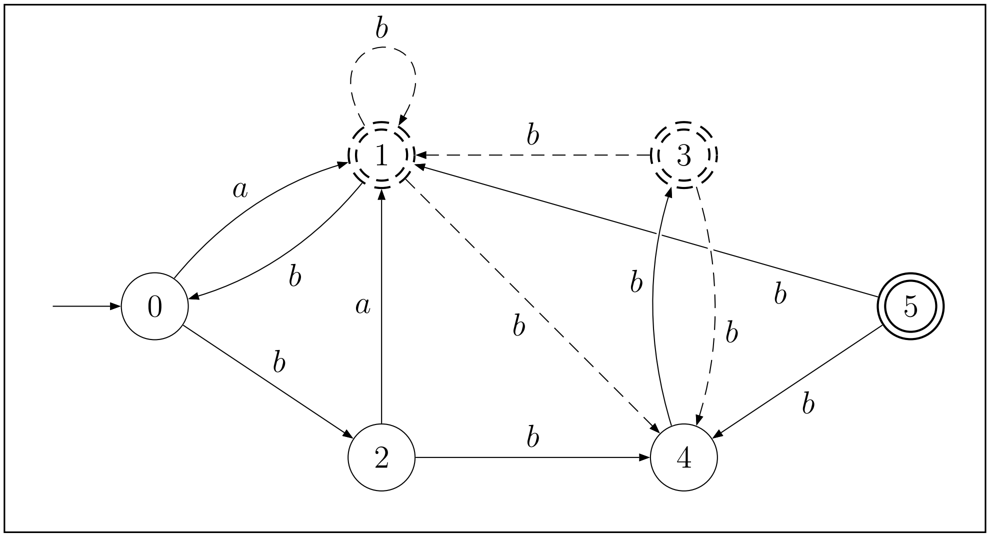
Question 5 - marks

Convert the the following ε-NFA into a DFA ($\sigma = \{a, b\}$). Intermediate steps are required.

ε-completion (added transitions are dashed).



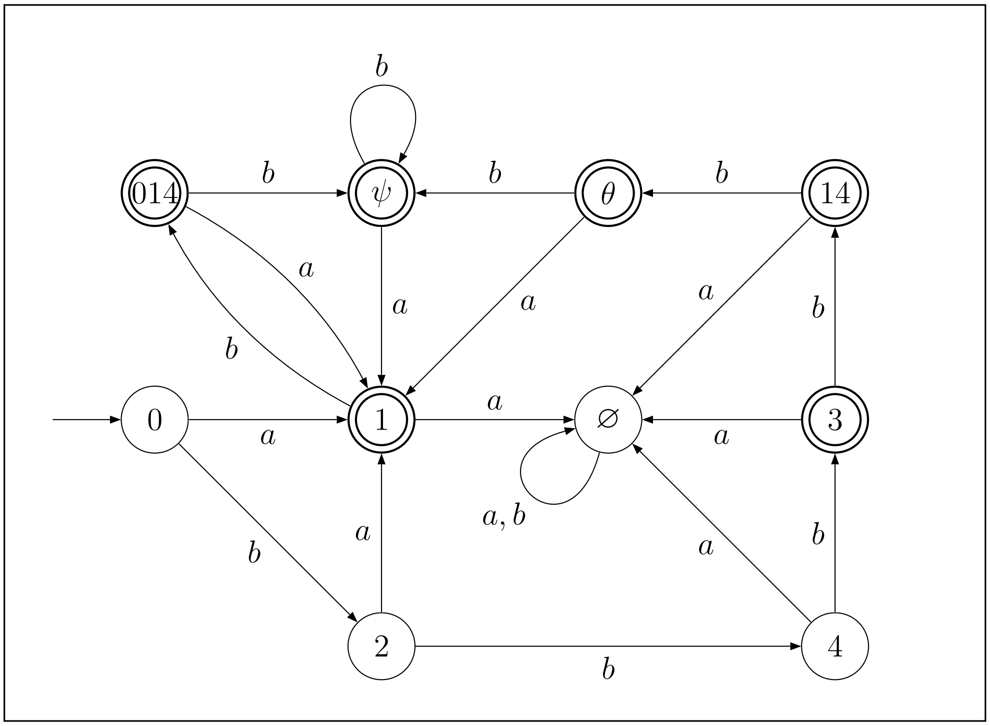
ε-removal (added transitions/final states are dashed).



NFA to DFA

We use the iterative subset construction (lecture notes page 65), sets in bold indicate first appearance. *This table is required for full credit*

states	a	b
$\{0\}$	$\{\mathbf{1}\}$	$\{\mathbf{2}\}$
$\{1\}$	$\varnothing$	$\{\mathbf{0}, \mathbf{1}, \mathbf{4}\}$
$\{2\}$	$\{1\}$	$\{\mathbf{4}\}$
$\varnothing$	$\varnothing$	$\varnothing$
$\{0, 1, 4\}$	$\{1\}$	$\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$
$\{4\}$	$\varnothing$	$\{\mathbf{3}\}$
$\psi = \{0, 1, 2, 3, 4\}$	$\{1\}$	$\{0, 1, 2, 3, 4\}$
$\{3\}$	$\varnothing$	$\{\mathbf{1}, \mathbf{4}\}$
$\{1, 4\}$	$\varnothing$	$\{\mathbf{0}, \mathbf{1}, \mathbf{3}, \mathbf{4}\}$
$\theta = \{0, 1, 3, 4\}$	$\{1\}$	$\{0, 1, 2, 3, 4\}$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