UNIVERSITY OF WESTERN ONTARIO

Computer Science 3331a, 2011

Foundations of Computer Science I

ASSIGNMENT 1

Due: Wednesday, September 28, 2011

1. Prove by induction on n that

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

2. Given an equivalence relation R over Σ and two arbitrary equivalence classes $[a]_R$ and $[b]_R$, where $a, b \in \Sigma$, prove that either $[a]_R = [b]_R$ or $[a]_R \cap [b]_R = \emptyset$.

3. Let $A = \{a, b, c, d, e\}$ be a set and $R = \{(a, b), (b, c), (c, d), (d, c)\}$ a relation defined on $A \times A$. Find the transitive closure and the reflexive and transitive closure of R.

4. Let $L = \{aa, abc, cba\}$ be a language (i.e., a set of words) over the alphabet $\Sigma = \{a, b, c\}$. Answer what the set of all subsets of L (i.e., the power set of L 2^L) is.

5. Give an English description for each of the following languages and answer whether the given word is in the language:

   (a) $L_1 = \{a^n b^n \mid n \geq 0\}$ and $x_1 = abab$

   (b) $L_2 = \{waa \mid w \in \{a, b\}^*\}$ and $x_2 = ababaa$

   (c) $L_3 = \{a^{2^n} \mid n \geq 0\}$ and $x_3 = aaaa$

6. Let $L_1 = \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$ and $L_2 = \{a^i b^j \mid i, j \geq 0\}$. Describe the language $L = L_1 \cap L_2$.