CS 3331a - Assignment 4 - Solutions

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Question 1

Given the NFA M, construct an equivalent regular grammar. (Sentence symbol = A).

$$
\begin{aligned}
A &\to aB \\
B &\to bD \mid aC \\
C &\to aE \mid bA \\
D &\to \varepsilon \mid aE \mid bF \mid cC \\
E &\to cG \\
F &\to aF \mid cG \\
G &\to \varepsilon \mid aC
\end{aligned}
$$

Question 2

Change the CFG into an equivalent CFG in Chomsky normal form:

1. Reduction

1i. Reduction : remove Nonterminating = $\{C\}$

$$
\begin{aligned}
S &\to aAB \mid aABD \\
A &\to aD \mid B \mid \varepsilon \\
D &\to AB \\
B &\to bA \mid \varepsilon \\
\not{C} &\to cC \\
E &\to cc \\
F &\to BBA \mid \varepsilon
\end{aligned}
$$

1ii. Reduction : remove Nonreachables $= \{E, F\}$

$$S \to aAB \mid aABD$$
$$A \to aD \mid B \mid \varepsilon$$
$$D \to AB$$
$$B \to bA \mid \varepsilon$$
$$\cancel{E} \to cc$$
$$\cancel{F} \to BBA \mid \varepsilon$$

2. Remove ε-productions

2i. Remove $B \to \varepsilon$ and reduce

$$S \to aAB \mid aA \mid aABD \mid aAD$$
$$A \to aD \mid B \mid \varepsilon \mid \cancel{\varepsilon}$$
$$D \to AB \mid A$$
$$B \to bA$$

2ii. Remove $A \to \varepsilon$

$$S \to aAB \mid aB \mid aA \mid a \mid aABD \mid aBD \mid aAD \mid aD$$
$$A \to aD \mid B$$
$$D \to AB \mid B \mid A \mid \varepsilon$$
$$B \to bA \mid b$$

2iii. Remove $D \to \varepsilon$ and reduce

$$S \to aAB \mid aB \mid aA \mid a \mid aABD \mid \cancel{aAB} \mid aBD \mid \cancel{aB} \mid aAD \mid \cancel{aA} \mid aD \mid \cancel{a}$$
$$A \to aD \mid a \mid B$$
$$D \to AB \mid B \mid A$$
$$B \to bA \mid b$$

3. Remove Units

Remove $A \to B$, $D \to B$

$$S \to aAB \mid aB \mid aA \mid a \mid aABD \mid aBD \mid aAD \mid aD$$
$$A \to aD \mid a \mid bA \mid b$$
$$D \to AB \mid bA \mid b \mid A$$
$$B \to bA \mid b$$

Remove $D \to A$ and reduce

$$
\begin{aligned}
S &\to aAB \mid aB \mid aA \mid a \mid aABD \mid aBD \mid aAD \mid aD \\
A &\to aD \mid a \mid bA \mid b \\
D &\to AB \mid bA \mid b \mid aD \mid a \mid \cancel{bA} \mid \cancel{b} \\
B &\to bA \mid b
\end{aligned}
$$

3. Remove Long Productions

$$
\begin{aligned}
S &\to aT \mid aB \mid aA \mid a \mid aU \mid aV \mid aW \mid aD \\
T &\to AB \\
U &\to TD \\
V &\to BD \\
W &\to AD \\
A &\to aD \mid a \mid bA \mid b \\
D &\to AB \mid bA \mid b \mid aD \mid a \\
B &\to bA \mid b
\end{aligned}
$$

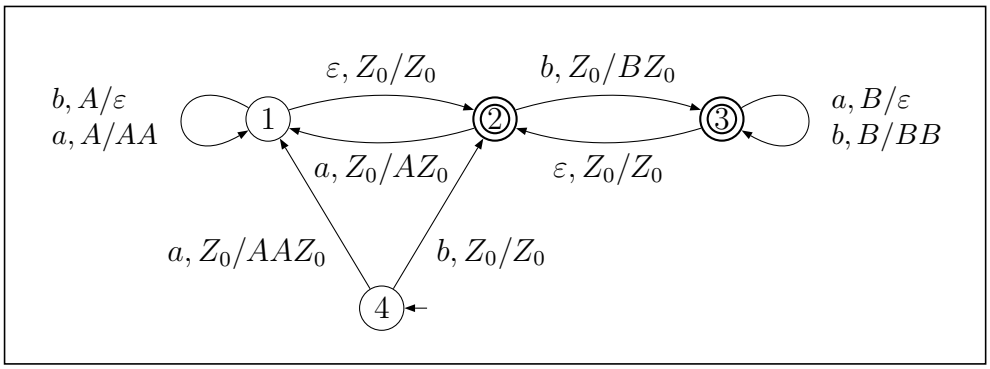
4. Convert to Chomsky Normal Form

$$
\begin{aligned}
S &\to \bar{a}T \mid \bar{a}B \mid \bar{a}A \mid a \mid \bar{a}U \mid \bar{a}V \mid \bar{a}W \mid \bar{a}D \\
T &\to AB \\
U &\to TD \\
V &\to BD \\
W &\to AD \\
A &\to \bar{a}D \mid a \mid \bar{b}A \mid b \\
D &\to AB \mid \bar{b}A \mid b \mid \bar{a}D \mid a \\
B &\to \bar{b}A \mid b \\
\bar{a} &\to a \\
\bar{b} &\to b
\end{aligned}
$$

Question 3

Construct pushdown automaton that accepts all words in $\{a, b\}^*$ such that the number of a's is (strictly) less than the number of b's.

The machine below accepts the required language by "final state".

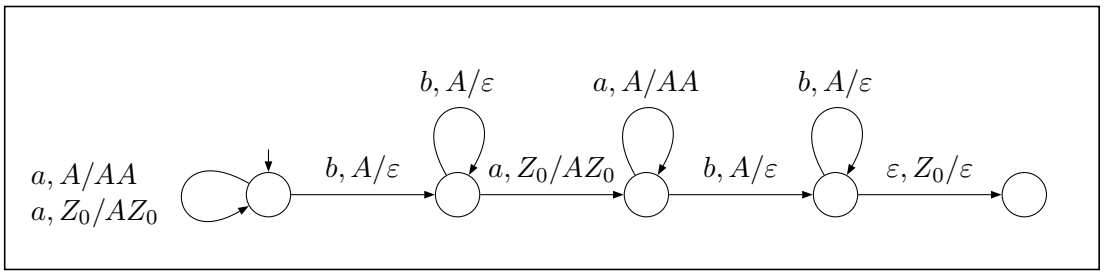


Note, in the above PDA I test that $|aw|_a \leq |aw|_b$. Thus, State 1 represents having read more a's than b's, State 2 an equal amount of a's and b's, State 3 more b's than a's and State 4 assumes that exactly one a has been read.

Question 4

Construct a deterministic pushdown automaton to accept the language $\{a^i b^i a^j b^j \mid i > 0, j > 0\}$.

The machine below accepts the required language by "empty stack".



Question 5 (Bonus)

Give a context-free grammar that generates the set of all words, over the alphabet $\Sigma = \{a, b\}$, that are not of the form ww for any word $w \in \Sigma^*$.

$$
\begin{aligned}
S &\to A \mid B \mid AB \mid BA \\
A &\to CAC \mid a \\
B &\to CBC \mid b \\
C &\to a \mid b
\end{aligned}
$$

Good luck on your exam everyone! Have a great holiday!

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