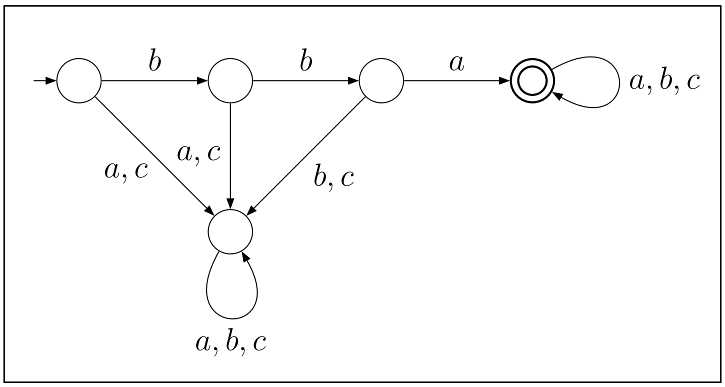
CS 3331a - Assignment 2 - Solutions

Paul Vrbik

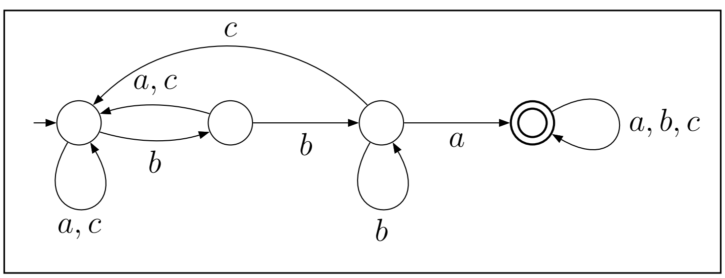
October 20, 2010

Question 1 - marks

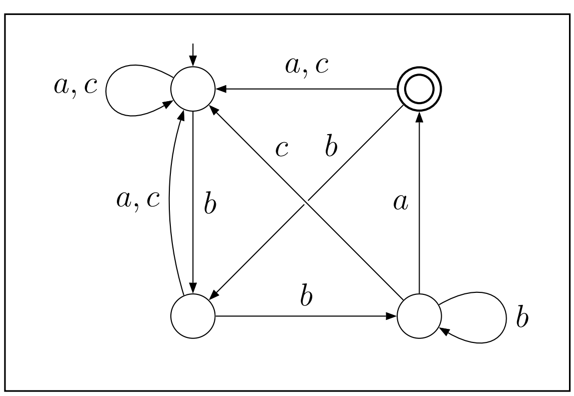
(1) The set of all words that have *bba* as a prefix.



(2) The set of all words that have *bba* as a subword.

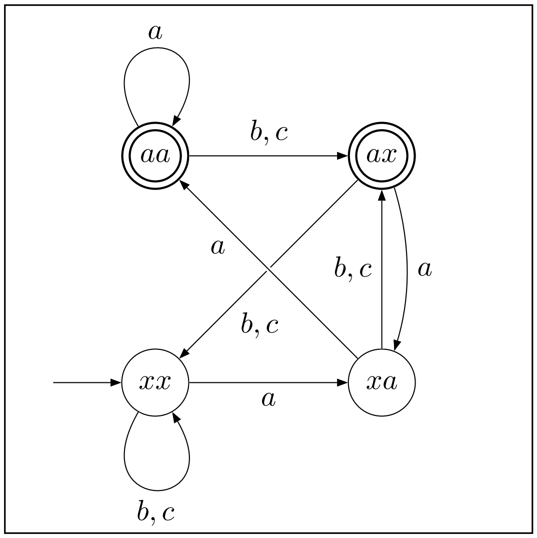


(3) The set of all words ending in *bba*.

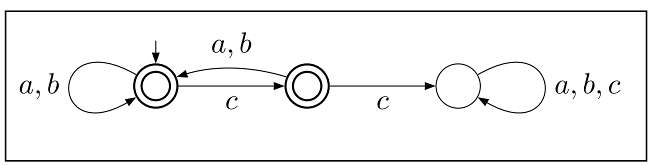


(4) The set of all words such that the second symbol from the right-end is a.

There is a strategy which may make this question easier: create states for all possible length two substrings (there are nine of them — but since we don't care to distinguish 'b' and 'c' we can represent both with 'x' and reduce to four substrings) and draw rules for transitioning between them. The accepting states are those substrings that begin with 'a'. (I've labelled the states below to reflect this.) You may interpret the start state as being 'bb' because you will at need at least two moves to get to an accepting state from there.



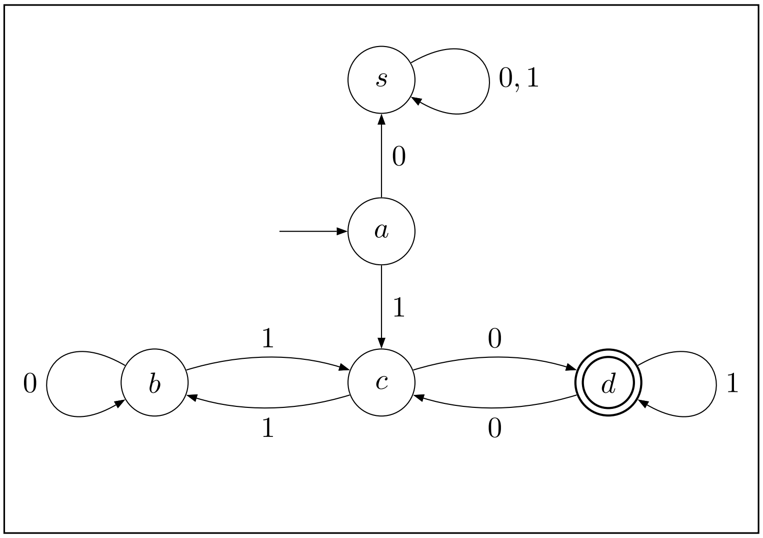
(5) The set of all words that do not contain consecutive c's.



Question 2 - marks

Give a full definition of a deterministic finite automaton (DFA) that accepts the set of all binary numbers (over the alphabet $\{0, 1\}$), each starting with 1 and the value of which is congruent to 2 modulo 3.

Note : in the diagram below states b, c and d denote 0 mod 3, 1 mod 3 and 2 mod 3 (respectively) and s is a sink state.



Let us call our DFA A. Using the "five-tuple" notation we have:

$$A = (\{a, b, c, d, s\}, \{0, 1\}, \delta, a, \{c\})$$

where $\delta : Q \times \Sigma \to Q$ is given by

$$\begin{array}{lllll} \delta(a,0) = s & \delta(b,0) = b & \delta(c,0) = d & \delta(d,0) = c & \delta(s,0) = s \\ \delta(a,1) = c & \delta(b,1) = c & \delta(c,1) = b & \delta(d,1) = d & \delta(s,1) = s\,. \end{array}$$

Question 3 - marks

Prove that the language $L = \left\{ a^i b^j \,\middle|\, 0 \le i < j \right\}$ is not accepted by *any* DFA.

Proof. Towards a contradiction assume there is a DFA

$$M = (Q, \{a, b\}, \delta, s, F)$$

such that $L = L(M)$. Let $n = \#Q$ (the number of states). Consider the accepting configuration for $a^n b^{n+1}$,

$$\underbrace{s_0 a^n b^{n+1} \vdash s_1 a^{n-1} b^{n+1} \vdash \cdots \vdash s_n b^{n+1}}_{n+1 \text{ states required}} \vdash \cdots \vdash s_{2n+1}$$

where $s_0 = s$ and $s_n \in F$. Note that $n+1$ states are required to read a^n (as indicated). However, by our assumption there is only n states; therefore by Pigeonhole principle two of these states must be the same. That is, there is $s_i = s_j$ such that $0 \le i < j \le n$.

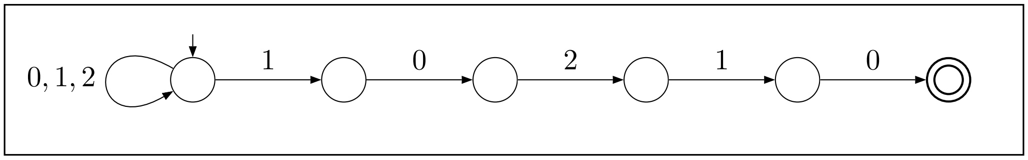
Now consider the accepting configuration,

$$\begin{aligned} s_0 a^{n+(j-i)} b^{n+1} &\vdash \cdots \vdash s_j a^{n-i} b^{n+1} \\ &\vdash s_i a^{n-i} b^{n+1} \vdash \cdots \vdash s_{2n+1} \end{aligned}$$

which shows that $a^{n+(j-i)} b^{n+1} \in L$ as well. Since $j - i > 0$ this is a contraction. □

Question 4 - marks

(1) The set of all words that have a suffix 10210.



(2) The set of all words such that the fourth symbol from the right-end is 0.