UNIVERSITY OF WESTERN ONTARIO

Computer Science 3331a, Fall 2009

Foundations of Computer Science I

ASSIGNMENT 4

Due: Monday, Dec. 7, 2009

No late assignment will be accepted

1. Give a regular grammar for each of the following languages:

   (1) $L_1 = \{0^i 1^j \mid i, j \geq 0\}$.

   (2) L_2 is the set of all words, over $\Sigma = \{a, b\}$, of even length.

   (3) L_3 is the set of all binary numbers divisible by 3.

2. Change the following CFG into an equivalent CFG in Chomsky normal form:

$$
\begin{aligned}
&S \to aAB \mid bBBC\\
&A \to aB \mid D \mid \varepsilon\\
&D \to AB\\
&B \to bA \mid \varepsilon\\
&C \to aC\\
&E \to cB\\
&F \to bAA \mid \varepsilon
\end{aligned}
$$

3. Construct a deterministic pushdown automaton that accepts all words in $\{a, b\}^*$ such that the number of a's is one less than the number of b's. (Both numbers are greater than or equal to zero.)

4. Construct a pushdown automaton to accept the language

$$\{a^i b^i \mid i > 0\} \cup \{a^i b^{2i} \mid i > 0\}.$$

5. Prove by using the pumping lemma for context-free languages that $L = \{ww \mid w \in \{0, 1\}^*\}$ is not a context-free language.

6. (Bonus) Give a context-free grammar that generates the set of all words, over the alphabet $\Sigma = \{a, b\}$, that are not of the form ww for any word $w \in \Sigma^*$.