UNIVERSITY OF WESTERN ONTARIO

Computer Science 3331a, 2010

Foundations of Computer Science I

ASSIGNMENT 1

Due: Wednesday, September 29, 2010

1. Given an equivalence relation R over Σ and two arbitrary equivalence classes $[a]_R$ and $[b]_R$, where $a, b \in \Sigma$, prove that either $[a]_R = [b]_R$ or $[a]_R \cap [b]_R = \emptyset$.

2. Let $A = \{1, 2, 3, 4, 5\}$ be a set and $R = \{(1, 3), (1, 4), (2, 5), (3, 2), (4, 1)\}$ a relation defined on $A \times A$. Find the transitive closure and the reflexive and transitive closure of R.

3. Prove by induction on n that

$$1 + 3 + \cdots + (2n - 1) = n^2$$

4. Give a brief English description for each of the following languages and answer whether each of the given words is in the language:
   (a) $L_1 = \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$. $x_1 = abbaab$, $x_2 = baababbba$.
   (b) If x is in L_2, then axb and bxa are in L_2; if x and y are in L_2, then xy is in L_2; and no other words are in L_2. $y_1 = bbbaaa$, $y_2 = baaababb$.
   (c) $L_3 = \{w \in \{0, 1\}^+ \mid$ the value of w as a binary number is divisible by $5\}$.
   $z_1 = 01010$, $z_2 = 1101$.

5. Prove that $(AB)^R = B^R A^R$ for arbitrary given languages A and B.