CS 2209b - Quiz 2 - Solutions

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Jan 31, 2011

Question 1

Prove or refute the following three arguments using only the S/I-rules.

a. *Proof.*

1	$\sim (A \supset \sim B)$	
2	C	
3	$\sim (B \bullet \sim D)$	
4	$\sim (D \bullet E)$	
5	assume: $\sim ((A \bullet C) \vee \sim E)$	
6	$\therefore \quad A$	NIF: 1
7	$\therefore \quad B$	NIF: 1
8	$\therefore \quad D$	CS: 4, 7
9	$\therefore \quad \sim E$	CS: 3, 8
10	$\therefore \quad \sim (A \bullet C)$	CS: 9, 5
11	$\therefore \quad \sim A$ contradiction	CS: 2, 10
12	$\therefore \quad ((A \bullet C) \bullet \sim E)$	Contradiction of: 5

□

b. *Proof.*

1	$((\sim S \bullet F) \vee X)$	
2	$(\sim X \vee \sim S)$	
3	$(\sim S \supset \sim (\sim X \supset N))$	
4	assume: $\sim (\sim S \bullet \sim X)$	
5	assume: $(\sim S \bullet F)$	
6	$\therefore \quad \sim S$	AND: 5
7	$\therefore \quad X$	AND: 4
8	$\therefore \quad \sim (\sim X \supset N)$	MP: 3, 6
9	$\therefore \quad \sim X$ contradiction	NIF: 8
10	$\therefore \quad \sim (\sim S \bullet F)$	Contradiction of: 5
11	$\therefore \quad X$	DS: 2, 10
12	$\therefore \quad \sim S$	DS: 1, 11
13	$\therefore \quad \sim (\sim X \supset N)$	MP: 3
14	$\therefore \quad \sim X$ contradiction	NIF: 13
15	$\therefore \quad (\sim S \bullet \sim X)$	Contradiction of: 4

□

c. *Proof.*

1	$(A \vee \sim (B \supset \sim C))$	
2	$(\sim (\sim A \bullet D) \supset \sim B)$	
3	$B \supset A$	
4	assume: $(\sim B \supset \sim A)$	
5	assume: A	
6	$\therefore \quad B$	MT: 4, 5
7	$\therefore \quad (\sim A \bullet D)$	MT: 2, 6
8	$\therefore \quad \sim A$ contradiction	AND: 7
9	$\therefore \quad \sim A$	Contradiction of: 4
10	$\therefore \quad \sim (B \supset \sim C)$	DS: 1, 9
11	$\therefore \quad B$	NIF: 10
12	$\therefore \quad A$ contradiction	MP: 3
13	$\therefore \quad \sim (\sim B \supset \sim A)$	Contradiction of: 4

□

Question 2

Translate the following into wffs, prove/refute the conclusion using only S/I-rules.

- If the world had a beginning in time and it didn't just pop into existence, then the world was caused by God.
- If the world was caused by God, then there is a God.
- $\therefore$ either ($\vee$) the world had no beginning in time, or it just popped into existence.

(Use B,P,C, and G).

Solution. Converting everything to WFFs we have

1	$((B \bullet \sim P) \supset C)$
2	$(C \supset G)$
3	$\therefore \quad (\sim B \vee P)$

Refute by truth-assignment. Consider the "refutation box"'

$$B = 1, P = 0, C = 1, G = 1$$

with

$$\Big(\Big(B^1 \bullet \sim P^0\Big) \supset C^1\Big) = 1$$
$$\Big(C^1 \supset G^1\Big) = 1$$
$$\Big(\sim B^1 \vee P^0\Big) = 0.$$

This shows that one can make all the premises true with false conclusion. Therefore we conclude that the original assertion is *invalid*.

■

Question 3

Four students are planning to travel to Europe in the summer...

1. If Jim does not go or Kate goes, then Michael will not go.
2. If Kate goes, then Michael will go.
3. If Nick does not go or Michael does not go, then Kate will go.

Prove: Michael will go.

Proof.

1	$((\sim J \vee K) \supset \sim M)$	
2	$((\sim N \vee \sim M) \supset K)$	
3	$(K \supset M)$	
4	assume: $\sim M$	
5	$\therefore \quad \sim K$	MT: 3, 4
6	$\therefore \quad \sim (\sim N \vee \sim M)$	MT: 3, 5
7	$\therefore \quad M$ contradiction	NOR: 6
8	$\therefore \quad M$	Contradiction of: 4

$\therefore$ Michael will go. □

Typeset by: Paul Vrbik and compiled on February 1, 2011