CS 2209b - Quiz 1 - Solutions

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Question 1

Translate the following into logic.

   a. If you are young, then you are strong but inconsiderate. (use Y, S, and I).

     Solution.
$$(Y \supset (S \bullet I))$$

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   b. You feel hungry if and only if you didn't have lunch. (use F and H).

     Solution.
$$(F \equiv \sim H)$$

     or
$$((F \supset \sim H) \bullet (\sim H \supset F))$$

         ■

   c. I will go shopping tomorrow if it is not rainy nor windy. (use G, R, and W).

     Solution.
$$((\sim R \bullet \sim W) \supset G)$$

     or
$$(\sim (R \vee W) \supset G)$$

         ■

Question 2

Calculate the truth value (show some intermediate steps).

a. If $A = 0$, $B = 1$, and $C = 0$, then $((\sim A \vee B) \supset C) = ?$

Solution.

$$
\begin{aligned}
((\sim A \vee B) \supset C) &\equiv ((\sim 0 \vee 1) \supset 0) \\
&\equiv ((1 \vee 1) \supset 0) \\
&\equiv (1 \supset 0) \\
&\equiv 0
\end{aligned}
$$

∎

b. If $A = 1$, $B = 0$, and $C = 1$ then $(A \supset (B \vee ((C \bullet \sim B) \supset \sim (\sim B \supset A)))) = ?$

Solution.

$$
\begin{aligned}
(A \supset (B \vee ((C \bullet \sim B) \supset \sim (\sim B \supset A)))) &\equiv (1 \supset (0 \vee ((1 \bullet \sim 0) \supset \sim (\sim 0 \supset 1)))) \\
&\equiv (1 \supset (0 \vee ((1 \bullet 1) \supset \sim (1 \supset 1)))) \\
&\equiv (1 \supset (0 \vee (1 \supset \sim 1))) \\
&\equiv (1 \supset (0 \vee (1 \supset 0))) \\
&\equiv (1 \supset (0 \vee 0)) \\
&\equiv (1 \supset 0) \\
&\equiv 0
\end{aligned}
$$

∎

c. If $A = ?$, $B = 1$ and $C = 0$ then $(A \supset (B \bullet \sim C)) = ?$

Solution.

$$
\begin{aligned}
((A \supset B) \bullet \sim C) &\equiv (? \supset (1 \bullet \sim 0)) \\
&\equiv (? \supset 1) \\
&\equiv 1
\end{aligned}
$$

∎

d. If $A = ?$, $B = ?$ and $C = ?$ then $((A \vee (\sim B \vee B)) \vee ((C \supset \sim A) \vee (A \bullet \sim A)))$

Solution.

$$
\begin{aligned}
((A \vee (\sim B \vee B)) \vee ((C \supset \sim A) \vee (A \bullet \sim A))) &\equiv ((A \vee 1) \vee ((C \supset \sim A) \vee 0)) \\
&\equiv (1 \vee (C \supset \sim A)) \\
&\equiv 1
\end{aligned}
$$

∎

Question 3

Use the truth-table test and truth-assignment to show the validity of the following two arguments (a total of four answers):

A note from the TA: There are some ambiguities associated with Question 3a. Firstly (and most disconcerting) is the textbook's usage of the word "either" to denote logical or ($\vee$). It is otherwise widely understood that "either or" corresponds to logical xor (exclusive or), a variant of $\vee$ which is true only when *exactly* one predicate is true (i.e. one or the other, but not both).

Most people answered using $\vee$—which is fine. However, the answer I give is what I feel is the best solution for this question.

a. Either I or you will go (to the meeting). You will go. Therefore, I will not go.

Solution. Let Y and I denote the propositions "you will go" and "I will go" (respectively). The conversion of the above statement into logic is:

$$((Y \vee I) \bullet \sim (Y \bullet I))$$

(i.e. You or I will go—but not both).

TRUTH-TABLE TEST:

Y I	$((Y \vee I) \bullet \sim (Y \bullet I))$,	Y $\therefore$	$\sim I$
0 0	0	0	1
0 1	1	0	0
1 0	1	1	1
1 1	0	1	0

TRUTH-ASSIGNMENT

$i.$

$$((Y \vee I) \bullet \sim (Y \bullet I)) = 1$$
$$Y = 1$$
$$\sim I = 0$$

$ii.$

$$((Y \vee I) \bullet \sim (Y \bullet I)) = 1$$
$$Y^1 = 1$$
$$\sim I^1 = 0$$

$iii.$

$$\left(\left(Y^1 \vee I^1\right) \bullet \sim \left(Y^1 \bullet I^1\right)\right) = 1$$
$$Y^1 = 1$$
$$\sim I^1 = 0$$

$iv.$

$$\left(\left(Y^1 \vee I^1\right) \bullet \sim \left(Y^1 \bullet I^1\right)\right) \neq 1$$
$$Y^1 = 1$$
$$\sim I^1 = 0$$

And so we conclude that the original assertion is valid.

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b. If you are a boy, you are brave or tall. You are not brave. You are not tall. Therefore, you are not a boy.

Solution. Let M, B and T denote (respectively) the propositions "you are a boy (male)", "you are brave" and "you are tall". The conversion of the above statement into logic is:

$$(M \supset (B \vee T))$$

(i.e. if you are male, then you are brave or tall).

TRUTH-TABLE TEST:

M B T	$(M \supset (B \vee T))$,	$(\sim B \bullet \sim T)$ $\therefore$	$\sim M$
0 0 0	1	1	1
0 0 1	1	0	1
0 1 0	1	0	1
0 1 1	1	0	1
1 0 0	0	1	0
1 0 1	1	0	0
1 1 0	1	0	0
1 1 1	1	0	0

TRUTH-ASSIGNMENT

i.
$$(M \supset (B \vee T)) = 1$$
$$(\sim B \bullet \sim T) = 1$$
$$\sim M = 0$$

ii.
$$(M \supset (B \vee T)) = 1$$
$$\left(\sim B^0 \bullet \sim T^0\right) = 1$$
$$\sim M^1 = 0$$

iii.
$$\left(M^1 \supset \left(B^0 \vee T^0\right)\right) = 1$$
$$\left(\sim B^0 \bullet \sim T^0\right) = 1$$
$$\sim M^1 = 0$$

iv.
$$\left(M^1 \supset \left(B^0 \vee T^0\right)\right) \neq 1$$
$$\left(\sim B^0 \bullet \sim T^0\right) = 1$$
$$\sim M^1 = 0$$

And so we conclude that the original assertion is valid. ∎