CS 2209b - Midterm - Partial Solutions

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Question 2

Show $\{\supset, 0\}$ is adequate provided that $\{\vee, \sim\}$ is adequate.

Proof. It is trivial to show that

$$(P \supset 0) \equiv \sim P.$$

We also know that $P \vee Q \equiv \sim P \supset Q$; or, using our rule for $\sim P$:

$$(P \supset 0) \supset Q \equiv P \vee Q.$$

Since we can write $\{\vee, \sim\}$ (which is adequate) using only $\{\supset, 0\}$—this must too be adequate. □

Question 3

Prove or refute using: a) truth table; b) S/I rules; c) resolution.

$$\begin{array}{r|l} 1 & \sim(\sim E \bullet J) \supset (\sim P \supset I) \\ 2 & M \bullet \sim J \\ \hline 3 & \therefore \quad (M \bullet \sim P) \end{array}$$

Truth-table.

EIJMP	$(\sim(\sim E \bullet J) \supset (\sim P \supset I))$	$(M \bullet \sim J)$	$\therefore$ $M \bullet P$
00010	0	1	0
00110	1	0	0
01010	**1**	**1**	**0**
01110	1	0	0
10010	0	1	0
10110	0	1	0
11010	**1**	**1**	**0**
11110	1	0	0

Therefore the argument is invalid.

Note: observe that the conclusion can be false only when M or P is false. It must be the case that M is true since $M \bullet \sim J$ must be true. So we must only check cases where $M = 1$ and $P = 0$ (significantly reducing the number of rows).

□

S/I Rules. One can easily extract a invalid row from the truth table (I chose the second last row) and construct the "refutation box":

$$E = 1, I = 1, J = 0, M = 1, P = 0$$

and provide the corresponding truth assignment:

TRUTH-ASSIGNMENT

i.
$$\begin{aligned} \sim (\sim E \bullet J) \supset (\sim P \supset I) &= 1 \\ M \bullet \sim J &= 1 \\ \therefore \quad (M \bullet \sim P) &= 0 \end{aligned}$$

ii.
$$\begin{aligned} \sim \left(\sim E^1 \bullet J^0\right) \supset \left(\sim P^0 \supset I^1\right) &= 1 \\ M^1 \bullet \sim J^0 &= 1 \\ \therefore \quad \left(M^1 \bullet \sim P^0\right) &= 0 \end{aligned}$$

iii.
$$\begin{aligned} 1 \supset (1 \supset 1) &= 1 \\ 1 \bullet \sim 1 &= 1 \\ \therefore \quad (1 \bullet 0) &= 0 \end{aligned}$$

iv. Refuted

□

Resolution Proof.
First we convert all premises into conjunctive normal form:

$$\begin{aligned} \sim (\sim E \bullet J) \supset (\sim P \supset I) &\equiv \sim\sim (\sim E \bullet J) \vee (\sim\sim P \vee I) \\ &\equiv (\sim E \bullet J) \vee (P \vee I) \\ &\equiv (\sim E \vee P \vee I) \bullet (J \vee P \vee I) \end{aligned}$$

which gives the clauses $\{\sim E, P, I\}$ and $\{J, P, I\}$.

The second premise is already in CNF and gives clauses: $\{M\}$ and $\{\sim J\}$.

The negation of the conclusion, $\sim (M \bullet \sim P) \equiv (\sim M \vee \sim P)$ giving the clause $\{\sim M, \sim P\}$.

1	$\{\sim E, P, I\}$	premise
2	$\{J, P, I\}$	premise
3	$\{M\}$	premise
4	$\{\sim J\}$	premise
5	$\{\sim M, \sim P\}$	negated goal
6	$\{\sim E, \sim M, I\}$	: 1, 5
7	$\{P, I\}$	: 2, 4
8	$\{J, \sim M, I\}$	: 2, 5
9	$\{\sim P\}$	: 3, 5
10	$\{\sim E, I\}$	: 3, 6
11	$\{J, I\}$	: 3, 8
12	$\{\sim M, I\}$	: 4, 8
13	$\{I\}$	: 4, 11
14	$\{\sim M, I\}$	: 5, 7
15	$\{I\}$	: 7, 9

We have exhausted all cancellations and therefore this argument is invalid.

□

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